A THEORETICAL STUDY ON THE URBAN HEAT ISLAND EFFECT

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ABSTRACT

The atmospheric thermodynamic equations are solved analytically to study the urban circulation caused by city heating. The perturbed flow and temperature fields are obtained and compared with observational facts from some urban boundary experiments. The agreement between the theoretical and observational results shows that this study may shed light on the physical characteristics of heat island circulation.

I. INTRODUCTION

Urban circulation is a mesoscale phenomenon caused by variety of factors. The changes of the flow fields over a city, for instance, result from the combination of increased roughness and decreased stability. But effects of these two factors cannot be separately estimated by observations. Using the method of numerical experiments, one can test the effect quantitatively for every factor. The simulations, however, are still not able to penetrate the physical characteristics of the relation between the phenomenon and the cause of its formation. Thus, in order to understand the essence of this phenomenon, it is still necessary to develop theoretical studies from different aspects.

Bornstein et al. (1972) analyzed the nocturnal wind field over New York City. They concluded that the city retarded airflow when upwind rural speeds were greater than 3.6 m/s, while for cases below this critical value urban speeds were greater. According to their analyses, strong flow is decelerated by increased roughness, but weak flow is accelerated due to the horizontal pressure gradient force associated with the urban heating. So the effect of heat island is best expressed in weak flow condition while the effect of city friction is best investigated under strong flow. In this study we will limit attention to the effect of urban boundary heating. Assuming a mean background wind speed to be 3 m/s, we can neglect the effect of roughness without missing the reality.

In the present paper, a linearized system of atmospheric thermodynamic equations is used to obtain an analytical solution of heat island circulation caused simply by the urban heating. The distributions of flow and temperature fields as well as their dependence on the mean wind speed and the stability of the background atmosphere are analyzed. The results are compared with the observational facts from field experiments. The agreement between the flow and temperature fields calculated by the model and those observed shows that this theoretical study may shed light on the characteristics of heat island and provide a clue to observational and numerical studies on this phenomenon.

II. THEORETICAL MODEL

In the log-pressure system, the vertical coordinate is defined as

$$z^* \equiv -H\ln\frac{p}{p_0},$$

where p_0 is the reference pressure (1000 hPa) and $H = RT_0/g$ is a constant scale height.

If the Coliolis and friction forces are neglected, the equations of horizontal momentum, hydrostatic, continuity and thermodynamic energy in the two-dimensional (x, z^*) plane can be expressed, respectively, as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w^*\frac{\partial u}{\partial z^*} = -\frac{\partial \Phi}{\partial x}, \quad (1)$$

$$\frac{\partial \Phi}{\partial z^*} = \frac{RT}{H}, \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w^*}{\partial z^*} - \frac{w^*}{H} = 0, \quad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w^*\Gamma = \frac{\varepsilon}{c_p}, \quad (4)$$

where $w^* = \frac{dz^*}{dt}$ is the vertical velocity in $(x,\ z^*)$ coordinate system; $\Gamma = \frac{\partial T}{\partial z^*} + \frac{RT}{c_pH} = \frac{T}{\theta}\frac{\partial \theta}{\partial z^*}$, which is nearly constant in the lower troposphere; ε is the heating rate per unit mass of air.

All the variables above can be represented as the sum of the averaged and the disturbed components

$$\psi = \bar{\psi} + \psi',$$

where ψ can be u, Φ, T, w^* and ε, and $\bar{w}^* = 0$. Using the representation above, we obtain the steady state, linearized perturbation equations from Eqs. (1)—(4)

$$\bar{u}\frac{\partial u'}{\partial x} + \frac{\partial \Phi'}{\partial x} = 0, \quad (5)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w^{*\prime}}{\partial z^*} - \frac{w^{*\prime}}{H} = 0, \quad (6)$$

$$\bar{u}\frac{\partial}{\partial x}\left(\frac{\partial \Phi'}{\partial z^*}\right) + w^{*\prime}S = \frac{\varepsilon' R}{c_pH}, \quad (7)$$

where $S = R\Gamma/H$ is a constant representing the atmospheric static stability. For convenience, we will omit the asterisk of the variables in the following representations.

Rearranging Eqs. (5)—(7), we have the equation of w

$$\frac{\partial^2 w}{\partial z^2} - \frac{1}{H}\frac{\partial w}{\partial z} + \frac{S}{\bar{u}^2}w = \frac{1}{\bar{u}^2}\frac{\varepsilon' R}{c_pH}, \quad (8)$$

where the inhomogeneous term on the right side indicates the heating effect of the urban boundary overlapping on the averaged atmospheric heating (or cooling) field. In a nighttime stable urban boundary layer, this term can be formally expressed as

$$\frac{1}{\bar{u}^2}\frac{\varepsilon' R}{c_p H}=A\left(\frac{2L^2}{x^2+L^2}-1\right)e^{-az}, \tag{9}$$

where L is the half-width length of the city, A is the heating effect at the surface of the city center which is set at $x=0$.

Eq. (8) can be solved analytically at given boundary conditions. The boundary conditions are set as follows:

$$w=0, \text{ when } z=0 \text{ (at the surface)} \tag{10}$$

and the vertical velocity has its maximum at a certain height $z=h$ in the lower troposphere

$$\frac{\partial w}{\partial z}=0. \tag{11}$$

Then the vertical velocity can be obtained as

$$w=\{e^{z/H}(-\cos\lambda z+B\sin\lambda z)+e^{-az}\}N, \tag{12}$$

where

$$N=\frac{A}{a^2+a/H+S/\bar{u}^2}\left(\frac{2L^2}{x^2+L^2}-1\right), \tag{13}$$

$$B=\frac{ae^{-ah}+\frac{1}{H}e^{h/H}\cos\lambda h-\lambda e^{h/H}\sin\lambda h}{e^{h/H}\left(\frac{1}{H}\sin\lambda h+\lambda\cos\lambda h\right)}, \tag{14}$$

$$\lambda=\sqrt{S/\bar{u}^2-1/4H^2}. \tag{15}$$

Substituting Eq. (12) into Eq. (6), we get the perturbed horizontal velocity u' as

$$u'=\left\{-e^{z/H}(\lambda\sin\lambda z+B\lambda\cos\lambda z)+\left(\frac{1}{H}+a\right)e^{-az}\right\}$$

$$\times\frac{A}{a^2+a/H+S/\bar{u}^2}\left(2L\arctan\frac{x}{L}-x\right)+C. \tag{16}$$

From Eq. (4) we can obtain the equation of the perturbed temperature T'

$$\bar{u}\frac{\partial T'}{\partial x}+w\Gamma=\varepsilon'/c_p, \tag{17}$$

which has the solution

$$T'=\frac{A}{\bar{u}}\left\{\frac{\bar{u}^2H}{R}e^{-az}-[e^{z/H}(-\cos\lambda z+B\sin\lambda z)+e^{-az}]\right.$$

$$\left.\times\frac{\Gamma}{a^2+a/H+S/\bar{u}^2}\right\}\left(2L\arctan\frac{x}{L}-x\right)+D. \tag{18}$$

the constants C and D in Eqs. (16) and (18) can be given by the values of the variables at specified positions. Assuming, for instance, $T'=0$ and $u'=0$ at $x=0$, we have $C=0$ $D=0$.

III. RESULTS AND DISCUSSION

In the lower atmosphere, the parameters are set as stability parameter $\Gamma = 0.006$ °C/m and mean wind speed $\bar{u} = 3$ m/s. Then we have $S = 0.0002$ s^{-2} and $\lambda = 0.0047$ m^{-1}. Constant α is assumed to be 0.004 m^{-1}, implying that at $z = 250$ m the heating rate in the urban boundary decreases to $1/e$ of its surface value. h is set to be 400 m, L 5000 m, and the perturbed heating rate at the surface of city center $\varepsilon' = 3$ J/(kg s).

Substituting above values into Eqs. (12), (16) and (18), we obtain the perturbation fields of flow and temperature $u'(x,z)$, $w(x,z)$ and $T'(x,z)$. Putting the perturbed quantities and the averaged ones together, we have the distributions of flow and temperature fields as $(\bar{u}+u'(x,z),\ w(x,z))$ and $(\bar{T}(z)+T'(x,z))$, shown in Figs. 1 and 2, respectively.

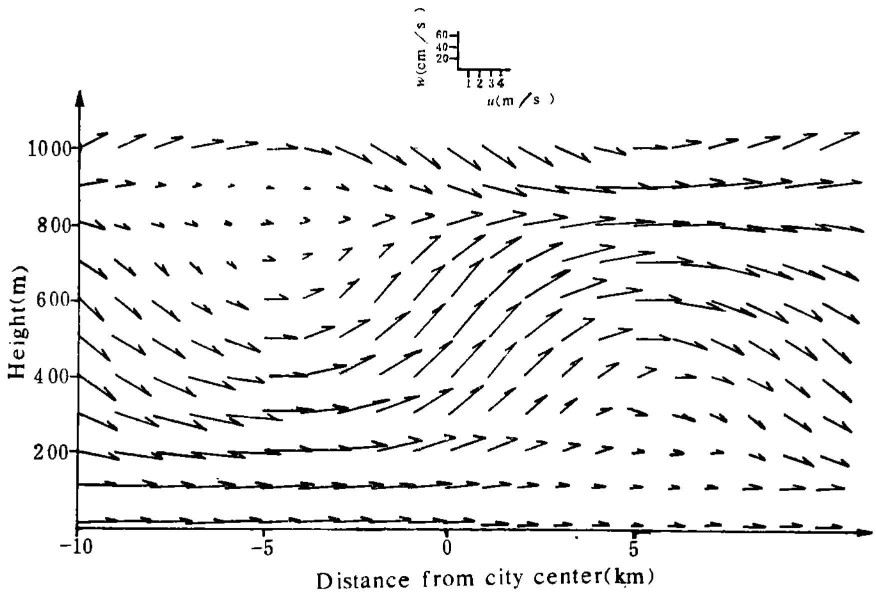

Fig. 1. Flow fields in the vertical cross-section.

Fig. 1 shows that flow over the urban area is convergent below the height h and divergent above h. The maximum perturbed velocities occur at the upwind and downwind edges of the city, $x = \pm L$ respectively. Then the area with the lowest horizontal velocity is found in the downwind part of the city. This sort of flow pattern has been observed in many cities. Pooler (1963), for instance, analyzed the flow fields of Louisville City. The streamlines given by him shows that in the stable atmosphere as air passes over a city the flow converges at the city center in most casses.

It is also shown in Fig.1 that there are updrafts in the lower boundary layer with maximum values at the city center. The flow fields over the city present wave-like motion which has its trough upwind of the urban area and its crest downwind. The phenomenon of wave-like motion over a city is called thermal-mountain wave since it is similar to the wave motion over a mountain ridge. Thermal-mountain wave can be detected indirectly by cloud observations over a city. Having analyzed the cloud data over Munich, Kratzer (1956) gave a cloud distribution model over a city, According to his model, most of the cloud can be found in the wake of the city. This coincides with the present model since city wake is just the place where the streamlines of the wave reach, as shown in Fig. 1, their maximum heights.

Since the perturbed velocities (12) and (16) possess wave-like characteristics, the composed wind speeds have rather complex pattern. As shown in Fig. 1, a low-level jet stream forms near the ground at the upwind edge, lifts to a height of about 300 m over the city center and finally reaches its maximum height over the downwind part of the city. If a non-uniform mean wind is considered, as in real situation, the urban wind speed profile will be more complex. Having analyzed wind data over St. Louis, Ackerman (1974) found that in light wind conditions at night a low-lever wind speed maximum often occurred in both urban and rural areas, but the urban one tended to appear at a higher elevation.

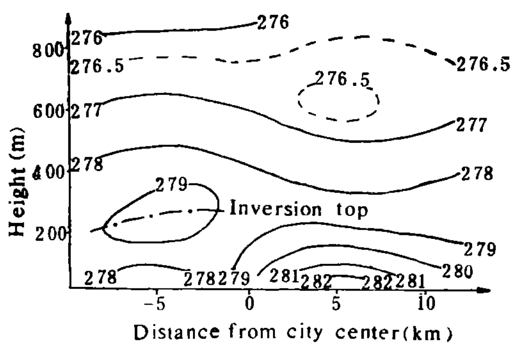

Fig. 2. The vertical cross-section of temperature distribution.

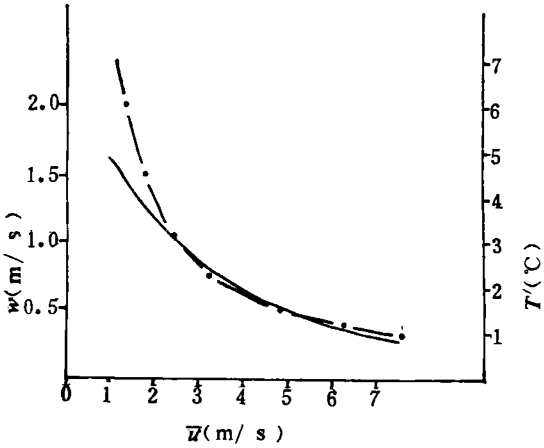

Fig. 3. Amplitudes of vertical velocity w at city center (solid line) and surface perturbed temperature T' at city upwind edge (dashed line) versus mean wind speed $\bar{u}$.

Fig. 2 shows the vertical section of the temperature distribution. The temperature maximum at surface, i.e. the core of heat island, occurs at the downwind part of the city. This agrees with the temperature distribution of Kumatany City, Japan analyzed by Kamura (1964). He argued that in light wind conditions (wind speed less than 5m/s) all of the centers of the highest surface temperature, without exception, appear at the downwind edge of the city. As the lapes rate over the city is larger than that over the suburb, the temperature above 300 m inside of the city is in turn lower than that outside. The temperature profiles over the city and over the suburb intersect at a height between 200-300 m. Duckworth and Sandberg (1954) analyzed the temperature profiles over downtown and suburb of some cities, such as San Francisco and Palo Alto, and found these so-called cross-over phenomena. They pointed out that the larger the cities, the clearer the phenomena.

It can be seen from Fig.2 that the temperature structure over the upwind suburb is stable. An inversion layer extends from the ground to a height of about 300 m. This inversion is caused by the warming of the descending wave motion. And the upper boundary layer over the downwind part of the city is an area with lower temperature induced by the ascending cooling. The temperature structure shown in Fig. 2 is similar to the temperature distributions of New York City at 0407—0612 on July 16, 1964 observed by Bornstein (1968). His statistics also showed that the frequency and strength of the ground inversion over city are less than that over the rural, but more for the elevated inversion. This conclusion agrees with the result of this model as shown in Fig. 2.

It is seen from (8) and (9) that the decrease of mean wind speed $\bar{u}$ will enhance the forced heating term on the right side of (8), i.e. A in (9), which plays a role as amplitudes for the perturbations u', T' and w, as shown in Eqs. (12), (16) and (18). Fig. 3 shows that as mean wind speed increases, all the perturbations of flow and temperature will be suppressed. As wind speed is larger than 6 m/s, the heat island circulation caused by thermal effects exists, in fact, neither in the model calculations nor in the real situations.

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