EFFECT OF THE THERMAL FORCING ON THE SECONDARY CIRCULATION OF TYPHOONS

Liu Yuezhen (刘月贞),

Institute of Atmospheric Physics, Academia Sinica, Beijing

Ding Yihui (丁一汇)

Academy of Meteorological Science, State Meteorological Administration, Beijing

and *Tao Shiyan* (陶诗言)

Institute of Atmospheric Physics, Academia Sinica, Beijing

Received May 11, 1989

ABSTRACT

A non-dimensional secondary circulation equation for typhoons has been derived and then 11-yr compositing typhoon data were used to estimate the thermally forced secondary circulation. The main results have been obtained as follows:

(1) The diabatic heating and Cu vertical heat mixing are major thermal forcing factors. They have the same magnitude of order. (2) The effects of eddy flux and Cu horizontal mixing of heat are of minor importance. (3) Ekman pumping and Cu vertical heat mixing cooperatively work. This feedback process is favorable for the enhancement of the secondary circulation of typhoons.

I. INTRODUCTION

There exists the apparent secondary circulation in numerous weather systems which plays an important role in maintenance and development of the systems. Eliassen (1951) first discussed the problem of secondary circulation systematically. Later, Sawyer (1956) and Eliassen (1962) derived the equations of the secondary circulation for front and jet stream-frontal system, independently. Shapiro (1981) derived a more complete equation of the secondary circulation with taking into account the effect of eddy vertical transport of momentum and heat, and diabatic heating. Recently, the secondary circulation for typhoons has been studied. For instance, Willoughby (1979) estimated the secondary circulation of tropical cyclones. Then, Shapiro and Willoughby (1982) set the forcing term to be a point source and estimated the secondary circulation forced by heat and momentum sources by using an ideal field.

The secondary circulation, relative to the basic circulation or primary circulation, is defined as a transverse circulation superposed on the basic circulation that is controlled by the physical process in the system (Holton, 1972). The basic circulation or primary circulation is a sort of circulation which satisfies some balance relationship (for example, geostrophic balance and gradient wind balance). Once this balance relationship of the primary circulation is destroyed, then the secondary circulation would be induced and acts as a

restoring mechanism of the equilibrium state of the basic flow. Therefore, the secondary circulation is a result of the continuous adjustment of the basic flow with a general tendency to maintain its balance relationship.

The physical factors affecting the secondary circulation of typhoons may be divided into two types: thermal forcing and dynamic forcing. The present paper will only deal with the former based on the observed data of typhoons. The discussion of the dynamic factors may refer to Sun and Ding (1989).

The thermal forcing includes the large-scale and Cu-scale condensation heating, Cu heat transport, the turbulent heat transport, radiative heating (or cooling) and re-evaporation in clouds. Among them, the release of the Cu convective condensation latent heat plays an important role in maintenance and development of typhoons. Cu heat vertical mixing and radiative heating have been stressed only recently. This paper will estimate the secondary circulations forced by various thermal factors and discuss their relative importance.

II. EQUATIONS

If one considers the set of primitive equations in the cylindrical coordinate system with $Z=-H\ln(p/p_0)$ as the vertical coordinate, its expressions are as follows:

$$\frac{\partial u}{\partial t}+\frac{u}{r}\frac{\partial u}{\partial \lambda}+u\frac{\partial u}{\partial r}-v\left(\frac{v}{r}+f\right)+w\frac{\partial u}{\partial z}+\frac{\partial \phi}{\partial r}=\Delta_a u, \tag{1a}$$

$$\frac{\partial v}{\partial t}+\frac{v}{r}\frac{\partial v}{\partial \lambda}+u\left(\frac{\partial v}{\partial r}+\frac{v}{r}+f\right)+w\frac{\partial v}{\partial z}+\frac{1}{r}\frac{\partial \phi}{\partial \lambda}=\Delta_a v, \tag{1b}$$

$$\frac{\partial w}{\partial t}+\frac{v}{r}\frac{\partial w}{\partial \lambda}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}-b+\frac{\partial \phi}{\partial z}=\Delta_a w, \tag{1c}$$

$$\frac{\partial b}{\partial t}+\frac{v}{r}\frac{\partial b}{\partial \lambda}+u\frac{\partial b}{\partial r}+N^2 w=\frac{g}{\theta_r}\frac{\theta}{T}Q+\Delta_a\theta, \tag{1d}$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \lambda}+\frac{\partial w}{\partial z}-\frac{w}{H}=0, \tag{1e}$$

where $b=g(T-T_r)/T_r$, buoyance force, $N^2=g(\gamma_d-\gamma)/T_r$, the squared buoyance frequency, ϕ is geopotential height, f is the Coriolis force, T and θ are temperature and potential temperature in cloud, respectively, T_r and θ_r are the environmental temperature and potential temperature, respectively, γ the temperature lapse rate, and γ_a the dry adiabatic lapse rate. Also, $\Delta_a u=k_H(\nabla^2-1/r^2)u+k_z\partial^2 u/\partial z^2$, $\Delta_a v=k_H(\nabla^2-1/r^2)v+k_z(\partial^2 v/\partial z^2)$, $\Delta_a w=k_H\left(\nabla^2-\frac{1}{r^2}\right)w+k_z\frac{\partial^2 w}{\partial z^2}$, $\Delta_a\theta=k_H\nabla^2\theta+k_z\frac{\partial^2\theta}{\partial z^2}$, $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{\partial}{\partial r}$ and H is the scale height, i. e., $H=RT_0/g_0$. Other symbols are conventional ones.

Averaging the equation set (1) over an appropriate area, one may obtain those terms which represent the effects of Cu convection:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \lambda}-v\left(f+\frac{v}{r}\right)+w\frac{\partial u}{\partial z}+\frac{\partial \phi}{\partial r}$$

$$=\Delta_a u-\left(\frac{1}{r}\frac{\partial \overline{u'v'}}{\partial \lambda}+\frac{\partial \overline{u'u'}}{\partial r}+\frac{\partial \overline{u'w'}}{\partial z}-\frac{\overline{v'v'}}{r}+\frac{\overline{u'u'}}{r}-\frac{\overline{u'w'}}{H}\right), \tag{2a}$$

$$\frac{\partial v}{\partial t}+\frac{v}{r}\frac{\partial v}{\partial \lambda}+u\left(\frac{\partial v}{\partial r}+\frac{v}{r}+f\right)+w\frac{\partial v}{\partial z}+\frac{1}{r}\frac{\partial \phi}{\partial \lambda}$$

$$=\Delta_a v-\left(\frac{\partial \overline{u'v'}}{\partial r}+\frac{1}{r}\frac{\partial \overline{v'v'}}{\partial \lambda}+\frac{\partial \overline{v'w'}}{\partial z}+\frac{2\overline{u'v'}}{r}-\frac{\overline{v'w'}}{H}\right), \tag{2b}$$

$$\frac{\partial w}{\partial t}+\frac{v}{r}\frac{\partial w}{\partial \lambda}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}-b+\frac{\partial \phi}{\partial z}$$

$$=\Delta_a w-\left(\frac{1}{r}\frac{\partial \overline{v'w'}}{\partial \lambda}+\frac{\partial \overline{u'w'}}{\partial r}+\frac{\partial \overline{w'w'}}{\partial z}+\frac{\overline{u'w'}}{r}-\frac{\overline{w'w'}}{H}\right), \tag{2c}$$

$$\frac{\partial b}{\partial t}+u\frac{\partial b}{\partial r}+\frac{v}{r}\frac{\partial b}{\partial \lambda}+N^2 w=\frac{g}{\theta_r}\frac{\theta}{T}Q+\Delta_a\theta$$

$$-\left(\frac{\partial \overline{b'u'}}{\partial r}+\frac{1}{r}\frac{\partial \overline{b'v'}}{\partial \lambda}+\frac{\partial \overline{b'w'}}{\partial z}+\frac{R}{c_p H}\overline{w'b'}+\frac{\overline{b'u'}}{r}-\frac{\overline{b'w'}}{H}\right), \tag{2d}$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \lambda}+\frac{\partial w}{\partial z}-\frac{w}{H}=0. \tag{2e}$$

In order to make the above set of equations dimensionless form, one may introduce the following variables:

$r=Rr^*, z=Zz^*, t=\frac{R}{v}t^*, u=Uu^*, v=Vv^*, w=Ww^*, \phi=(\mu gH/z)b^*, \rho(z)=(1.1$ kg m^{-3})$\rho^*(z)$, $H=\frac{z}{2}H^*, N=N^*n, f=5.0\times10^{-5}\mathrm{s}^{-1}$, absolute vorticity: $\zeta=\frac{v}{R}\left(\frac{\partial v^*}{\partial r^*}+\frac{v^*}{r^*}+\frac{1}{R_0}\right)=\frac{v}{R}\xi^*$, inertial parameter: $\zeta=\frac{v}{R}\left(\frac{2v^*}{r^*}+\frac{1}{R_0}\right)\frac{v}{R}\zeta^*$, vertical shear: $s=\frac{v}{z}\times\frac{\partial v^*}{\partial z^*}=\frac{v}{z}s^*$, Rossby number: $\mathrm{Ro}=\frac{v}{fR}$, Froude number: $\mathrm{Fr}=\frac{v^2}{\mu gH}$, Richardson number: $\mathrm{Ri}=\left(\frac{Nz}{v}\right)^2$, aspect ratio: $A^*=z/R$, and asterisk * denotes dimensionless quantities.

Introducing a small parameter, now we expand each variable in terms of the small parameter $\varepsilon=U/V$:

$$\begin{cases} u^*=u_s^*(r^*,z^*,\varepsilon t^*)+u_a^*(r^*,\lambda,z^*,t^*)+\varepsilon\cdots, \\ v^*=v_s^*(r^*,z^*,\varepsilon t^*)+\varepsilon v_a^*(r^*,\lambda,z^*,t^*)+\varepsilon^2\cdots, \\ w^*=w_s^*(r^*,z^*,\varepsilon t^*)+w_a^*(r^*,\lambda,z^*,t^*)+\varepsilon\cdots, \\ b^*=b_s^*(r^*,z^*,\varepsilon t^*)+\varepsilon b_a^*(r^*,\lambda,z^*,t^*)+\varepsilon^2\cdots, \\ \phi^*=\phi_s^*(r^*,z^*,\varepsilon t^*)+\varepsilon\phi_a^*(r^*,\lambda,z^*,t^*)+\varepsilon^2\cdots. \end{cases} \tag{3}$$

The subscript s denotes the symmetric part of physical quantities, while the subscript a represents asymmetric part. Each symmetric part delays its spatial scale.

Substituting the expressions into the set of equations (2), and taking the terms of order of ε' and averaging along λ direction which is expressed with [ ], one may obtain

$$v_s^*\xi_s^*=\frac{1}{F_r}\frac{\partial \phi_s^*}{\partial r^*}, \tag{4a}$$

$$\frac{\partial v_s^*}{\partial t^*}+u_s^*\xi_s^*+w_s^*s_s^*=k_H^*\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r^*}\frac{\partial}{\partial r^*}-\frac{1}{r^{*2}}\right)v_s^*+k_z^*\frac{\partial^2 v_s^*}{\partial z^{*2}}$$

$$-\frac{1}{r^{*2}}\frac{\partial r^{*2}\overline{u_s^{*\prime}v_s^{*\prime}}}{\partial r}-\frac{\partial\overline{v_s^{*\prime}w_s^{*\prime}}}{\partial z^*}, \tag{4b}$$

$$\frac{\partial\phi_s^*}{\partial z^*}=b_s^*, \tag{4c}$$

$$\frac{1}{F_r}\frac{\partial b_s^*}{\partial t^*}+u_s^*\xi_s^*s_s^*+\mathrm{Ri}N^{*2}w_s^*=\bar{Q}+\frac{1}{F_r}k_H^*\left(\frac{\partial^2}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\right)b_s^*$$

$$+\mathrm{Ri}k_z^*\frac{\partial^2 b_s^*}{\partial z^{*2}}-\frac{1}{F_r}\frac{\partial\overline{b_s^{*\prime}u_s^{*\prime}}}{\partial r^*}-\mathrm{Ri}\frac{\partial\overline{b_s^{*\prime}w_s^{*\prime}}}{\partial z^*} \tag{4d}$$

$$\frac{\partial u_s^*}{\partial r^*}+\frac{u_s^*}{r^*}+\frac{\partial w_s^*}{\partial z^*}-\frac{zw_s^*}{H}=0, \tag{4e}$$

where $\bar{Q}=\frac{1}{\varepsilon}\frac{Rz}{v^3}\frac{g}{\theta_r}\frac{\theta}{T}Q$.

The set of equation (4) is apparently the symmetric equation set to describe the tropical cyclone. From this set of equation, we can derive a diagnostic equation of the secondary circulation for the tropical cyclone:

$$\frac{\partial}{\partial r^*}\left(\frac{\mathrm{Ri}N^{*2}}{r^*\rho^*}\frac{\partial\psi}{\partial r^*}-\frac{s_s^*\xi_s^*}{r^*\rho^*}\frac{\partial\psi}{\partial z^*}\right)+\frac{\partial}{\partial z^*}\left(\frac{\zeta_s^*\xi_s^*}{r^*\rho^*}\frac{\partial\psi}{\partial z^*}-\frac{s_s^*\xi_s^*}{r^*\rho^*}\frac{\partial\psi}{\partial r^*}\right)$$

$$=\frac{\partial}{\partial r^*}\left[\bar{Q}+\frac{1}{F_r}k_H^*\frac{g^*}{\theta_r^*}\left(\frac{\partial^2}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\right)\theta_s^*+\mathrm{Ri}k_z^*\frac{g^*}{\theta_r^*}\frac{\partial^2\theta_s^*}{\partial z^{*2}}\right]$$

$$-\frac{\partial}{\partial r^*}\left(\frac{1}{F_r}\frac{g^*}{\theta_r^*}\frac{\partial\overline{\theta_s^{*\prime}u_s^{*\prime}}}{\partial r^*}+\mathrm{Ri}\frac{g^*}{\theta_r^*}\frac{\partial\overline{\theta_s^{*\prime}w_s^{*\prime}}}{\partial z^*}\right)-\frac{\partial}{\partial z^*}\left[\xi_s^*k_H^*\left(\frac{\partial^2}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\right.\right.$$

$$\left.\left.-\frac{1}{r^{*2}}\right)v_s^*+\xi_s^*k_z^*\frac{\partial^2 v_s^*}{\partial z^{*2}}\right]+\frac{\partial}{\partial z^*}\left[\xi_s^*\left(\frac{1}{r^{*2}}\frac{\partial r^{*2}\overline{u_s^{*\prime}v_s^{*\prime}}}{\partial r^*}+\frac{\partial\overline{v_s^{*\prime}w_s^{*\prime}}}{\partial z^*}\right)\right], \tag{5}$$

where ψ is stream function, $u_s^*=-\frac{1}{r^*\rho^*}\frac{\partial\psi}{\partial z^*}$ and $w_s^*=\frac{1}{r^*\rho^*}\frac{\partial\psi}{\partial r^*}$. While introducing the above stream function, the isothermal atmosphere with the temperature T_o is assumed. In this case, the height z defined as $-H\ln(P/P_0)$ is just equal to the actual height.

The elliptic condition for Eq. (5) is $D^*=\mathrm{Ri}N^{*2}\zeta_s^*\xi_s^*-(s_s^*\xi_s^*)^2$. The observed data were used to estimate D^* and we find that all the grid points of the domain under study satisfy the elliptic condition of $D^*>0$.

Neglecting the terms related to dynamic forcing in Eq. (5), one may have

$$\frac{\partial}{\partial r^*}\left(\frac{\mathrm{Ri}N^{*2}}{r^*\rho^*}\frac{\partial\psi}{\partial r^*}-\frac{s_s^*\xi_s^*}{r^*\rho^*}\frac{\partial\psi}{\partial z^*}\right)+\frac{\partial}{\partial z^*}\left(\frac{\zeta_s^*\xi_s^*}{r^*\rho^*}\frac{\partial\psi}{\partial z^*}-\frac{s_s^*\xi_s^*}{r^*\rho^*}\frac{\partial\psi}{\partial r^*}\right)$$

$$=\frac{\partial}{\partial r^*}\left[\bar{Q}+\frac{1}{F_r}k_H^*\frac{g^*}{\theta_r^{*2}}\left(\frac{\partial^2}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\right)\theta_s^*+\mathrm{Ri}k_z^*\frac{g^*}{\theta_r^*}\frac{\partial^2\theta_s^*}{\partial z^{*2}}\right]$$

$$-\frac{\partial}{\partial r^*}\left(\frac{1}{F_r}\frac{g^*}{\theta_r^*}\frac{\partial\overline{\theta_s^{*\prime}u_s^{*\prime}}}{\partial r^*}+\mathrm{Ri}\frac{g^*}{\theta_r^*}\frac{\partial\overline{\theta_s^{*\prime}w_s^{*\prime}}}{\partial z^*}\right), \tag{6}$$

Eq. (5) will be used to estimate the secondary circulation associated with the tropical

cyclone. The terms on the right hand of this equation represent thermal forcing for the secondary circulation, including the diabatic term, the horizontal and vertical turbulent flux of heat, and the Cu horizontal and vertical flux of heat. It is very interesting to evaluate their relative importance for forcing the secondary circulation in the tropical cyclone.

III. DATA AND COMPUTIONAL ASPECT

The data used in the present study are taken from the data set of compositing typhoons for 1966—1977 compiled by Prof. Gray's group of the Department of Atmospheric Sciences, Colorado State University. The characteristic values of the compositing typhoons are given below: V—30 m/s, U—10 m/s, W—1.4 m/s, R—111 km, $Z=15$km and ψ—1.83×10^7 t/s.

The ten layers in vertical are taken from 1000 hPa to 100 hPa at vertical intervals of 100 hPa. Table 1 shows Z^* values corresponding to each pressure level. It may be seen that the vertical resolution in terms of Z^* is variable.

The horizontal grid length $\Delta r^*=0.5$, i. e., half a latitude degree. Along the horizontal direction, we have 14 grid points. In calculation, the boundary coudition of $\psi=0$ was taken. This assumption of boundary condition may cause some uncertainty for the computational results.

Table 1. Z^* Values Corresponding to Each Pressure Level

Level	1	2	3	4	5	6	7	8	9	10
Pressure Level (hPa)	100	200	300	400	500	600	700	800	900	1000
Z^*	1.23	0.86	0.64	0.49	0.37	0.27	0.19	0.12	0.06	0.00

IV. RESULTS AND COMPARISON OF RELATIVE IMPORTANCE OF VARIOUS FORCING TERMS

1. *Diabatic Heating*

The diabatic heating term is here taken to be $Q=Q_L+Q_C+Q_R$, where Q_L is large-scale condensation heating, Q_C convective heating and Q_R the radiative heating (cooling). The sensible heat flux from the underlying surface is neglected.

Q_L is estimated by the following expressions

$$Q_L=-L\omega\frac{\partial q_s}{\partial p},\quad \frac{\partial q_s}{\partial p}=\frac{C_pC_s(1-pc_6)}{p(C_s-c_3c_5L)},$$

$$c_3=\frac{a_0}{T-b_0}-\frac{a_0(T-273.16)}{(T-b_0)^2},\quad c_5=\frac{-0.622\times6.11}{p}\exp\left[\frac{a_0(T-273.16)}{T-b_0}\right],$$

$$c_6=\left[\frac{a_0}{T-b_0}-\frac{a_0(T-273.16)}{(T-b_0)^2}\right]\left[\frac{RT}{C_pp}(1+0.61q_s)\right],$$

where L is condensation latent heat, q_s the saturation specific humidity and a_0, b_0 are the known constants.

The modified Kuo-scheme of Cu parameterization worked out by Krishnamurti et al. (1984) is used to estimate Q_C. This multiregression scheme has taken into account the reasonable partition between heating and moistening as well as meso-and small-scale moisture

convergence. No direct estimate of Q_R in the present paper has been done and only its climatological values were taken. On the average the mean cooling rate in clear region is -2— -3 °C/d while that in cloudy region is -1—1 °C/d. The diabatic heating may force a very strong secondary circulation (Fig. 1). Fig. 1a is the distribution of $\psi(\times 10^{-2})$, showing the center of the positive circulation cell located at 200 hPa at a distance of 270 km from the center of cyclone. The field of radial velocity u^* ($\times 10^{-1}$) indicates the inflow below 300 hPa and outflow above 300 hPa (Fig. 1b). The corresponding w^* ($\times 10^{-2}$) field indiacates the upward motion inward from $r^*=2$, 5 and the downward motion outward from $r^*=3.0$, with a transition zone found in 2.5—3.0 (Fig. 1c). In addition, at $r^*=5$ and 800 hPa, there is a negative circulation. It is not clear about the cause for its formation yet.

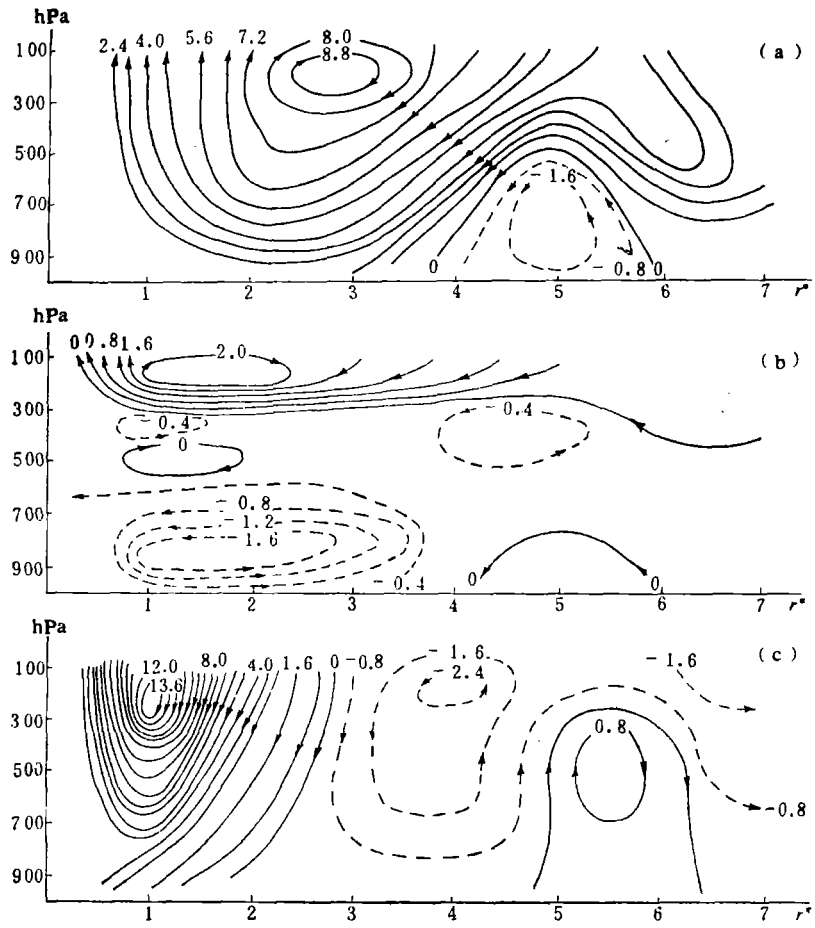

Fig. 1. The secondary circulation forced by the diabatic heating term: (a) the pattern of ψ, with interval of stream function 0.8×10^{-2}; (b) the pattern of the radial wind velocity u^*, with interval being 0.4×10^{-1} and solid lines denoting inflow and dashed lines outflow; and (c) the pattern of the vertical velocity w^*, with interval being 0.8×10^{-2}, and solid lines indicating the upward motion and dashed lines downward motion. The sense of arrows shows the direction of flow.

2. *Turbulent Flux of Heat*

This section will deal with the horizontal and vertical turbulent fluxes of heat. It is generally accepted that the vertical flux of heat is much more important than the horizontal flux in the development of typhoons. The expression of these two terms is as follows:

$$\frac{\partial}{\partial r^*}\left[\frac{k_H^* g^*}{F_r \theta_r^*}\left(\frac{\partial^2}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\right)\theta_s^*\right]+\frac{\partial}{\partial r^*}\left(\mathrm{Ri}\, k_H^*, k_z^* \frac{g^*}{\theta_r^*}\frac{\partial^2 \theta_s^*}{\partial z^{*2}}\right),$$

where k_H^* and k_z^* are non-dimensional horizontal and vertical eddy viscosity coefficient respectively, which are here taken to be 2.5×10^{-3} for k_H^* and 0.24×10^{-3} for k_z^*.

Fig. 2 shows the secondary circulation forced by the horizontal turbulent flux of heat. There is a weak positive circulation with two centers at about 750 hPa in the lower and middle troposphere. A negative circulation is located in the upper troposphere, with the two stronger centers found in the layer of 300—200 hPa.

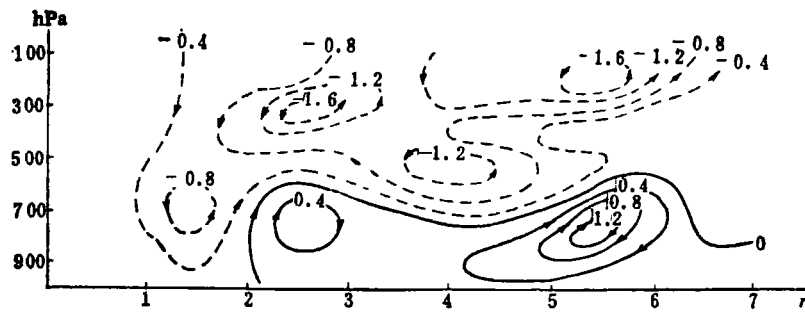

Fig. 2. The secondary circulation forced by the horizontal turbulent flux of heat (interval of ψ is 0.4×10^{-7}, and sense of arrows indicates the direction of flow).

Fig. 3 is the secondary circulation forced by the vertical turbulent flux of heat, showing a complete pair of positive and negative circulation cells, with the center of the former found at 550 hPa and the center of the latter at 750 hPa. Above 300 hPa, one may observe one weak positive circulation.

In comparison, the magnitude of secondary circulation forced by the vertical turbulent flux of heat is much greater than that forced by the horizontal turbulent flux of heat.

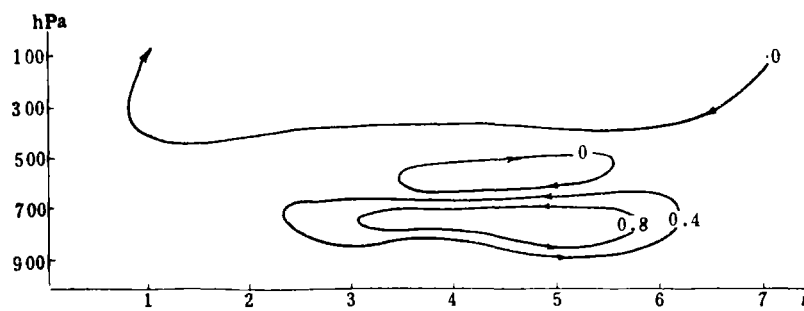

Fig. 3. The secondary circulation forced by the vertical turbulent flux of heat (interval of ψ is 0.4×10^{-4}).

3. *Cu Horizontal Flux of Heat*

It is generally recongnized that the transport of heat by Cu convection is much greater than that by turbulence. The Cu horizontal flux of heat is expressed by the following relation:

$$-\frac{\partial}{\partial r^*}\left(\frac{1}{F_r}\frac{g^*}{\theta_r^*}\frac{\partial\overline{\theta^{*\prime}u_s^{*\prime}}}{\partial r^*}\right).$$

Kuo (1974) suggested that this term may be estimated according to K-theory, but it is very important to appropriately take k_H value. So the above expression may be rewritten as $\frac{\partial}{\partial r^*}\left(\frac{1}{\mathrm{Fr}}\frac{g^*}{\theta_r^*}k_{Hc}^*\left[\left(\frac{\partial^2}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\right)\theta_s^*\right]\right)$. Here k_{Hc}^* is Cu eddy viscosity coefficient. It is very difficult to exactly determine the value of this coefficient. As a rough approximation, we take this coefficient to be as three times as the eddy viscosity coefficient i. e., $k_{Hc}^*=7.5\times10^{-3}$.

The secondary circulation thus obtained has a similar pattern to Fig. 2, only with greater magnitudes of ψ (not shown).

4. *Cu Vertical Flux of Heat*

The expression of Cu vertical flux of heat is

$$-\frac{\partial}{\partial r^*}\left(\mathrm{Ri}\frac{g^*}{\theta_r^*}\partial\overline{\theta_s^{*\prime}w_s^{*\prime}}/\partial z^*\right).$$

Based on the parameterization scheme proposed by Schneider and Lindzen (1976), the above expression may be further rewritten as follows:

$$-\frac{\partial}{\partial r^*}\left[\mathrm{Ri}\frac{g^*}{\theta_r^*}\frac{\partial}{\partial z^*}\left(\frac{M_C^*}{\rho^*}(\theta_s^*-\theta_{sc}^*)\right]\right.,$$

where M_C^* is the dimensionless Cu mass flux which is a crucial parameter to determine the Cu vertical mixing. In the following section, we shall discuss the computational aspect of M_C.

Fig. 4 is the secondary circulation forced by this term with M_C taken in accordance with $f_1(p)$, as will be discussed in the fallowing section. There is a complete positive circulation in the entire domain, with the center located at 300 km and 500 hPa.

5. *Comparison among Various Forcing Terms*

The intercomparison of various forcing terms has indicated that for the thermal forcing of the secondary circulation of typhoons the horizontal turbulent flux of heat is insignificant; the vertical turbulent flux of heat is relatively large; the Cu horizontal flux of heat is relatively small, only with the same magnitude of order as the horizontal turbulent flux of heat, while the Cu vertical flux of heat is more significant, with the magnitude of order of 10^{-2} and the rather complete circulation cell.

The diabatic heating is the primary forcing factor among them, with the magnitude of order of the secondary circulation being 10^{-2} and a greater magnitude of ψ than that for Cu vertical flux of heat. This vigorous positive circulation dominates nearly the entire domain under study.

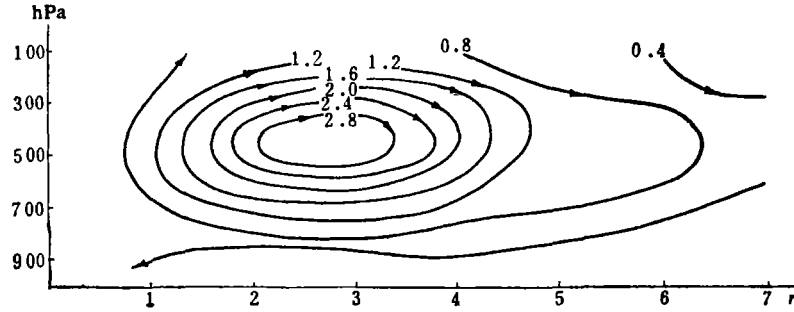

Fig. 4. The secondary circulation forced by Cu vertical flux of heat (interval of ψ is 0.4×10^{-2}).

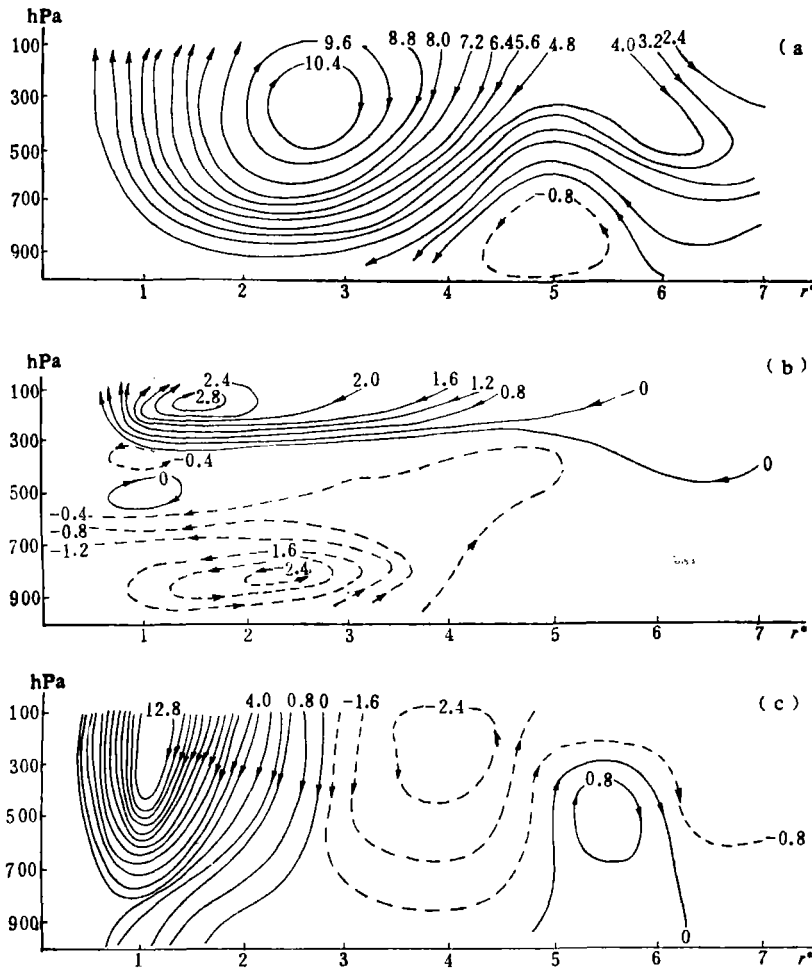

Fig. 5. The secondary circulation forced by all the thermal factors (five terms): (a) ψ-pattern with interval being 0.8×10^{-2}; (b) u^*-pattern, with interval being 0.4×10^{-1}; and solid lines indicating outflow and dashed lines inflow; and (c) w^*-pattern with interval being 0.8×10^{-1} and solid lines indicating upward motion and the dashed lines downward motion.

Fig. 5 is the secondary circulation forced by all the thermal forcing terms (including the above five factors), showing the similar feature to Fig. 1. A positive circulation dominates the entire domain with a weak negative circulation found at low level in the outer region of typhoon, indicating the primary importance of diabatic heating. The difference between Fig. 4 and Fig. 1 shows up in the enhancement of the positive circulation, the descent of position of circulation center and weakening of the negative circulation of the former, implying the neglegible role of Cu vertical mixing of heat. Therefore, the thermally forced secondary circulation is mainly controlled by these two terms.

V. Cu MASS FLUX M_C

In the following we shall discuss the estimate of M_C according to the scheme of Lindzen (1981). M_C has the following implicit expression:

$$\int M_C C_P (p/p_0)^{R/C_P} \frac{\partial \theta}{\partial p} dp = L p_r,$$

where p_r is precipitation rate. To determine the vertical profile of M_c, we assume

$$M_C = M_C(\max) \times f(p),$$

$$f_1(p) = \left[\exp\left(-\left(\frac{p-550}{250}\right)^2\right) - \exp\left(-\left(\frac{450}{250}\right)^2\right)\right] \times \left[1 - \exp\left(-\left(\frac{450}{250}\right)^2\right)\right]^{-1}.$$

$f_1(p)$ thus determined has the maximum unity (Fig. 6) at 550 hPa and zero values at 1000 and 100 hPa, respectively.

Here, $M_C(\max) = L p_r \Big/ \int f(p) C_P (p/p_0)^{R/C_P} \left(\frac{\partial \theta}{\partial p}\right) dp$. M_C derived from Exp $f_1(p)$ implies the presence of a layer of entrainment below 550 hPa.

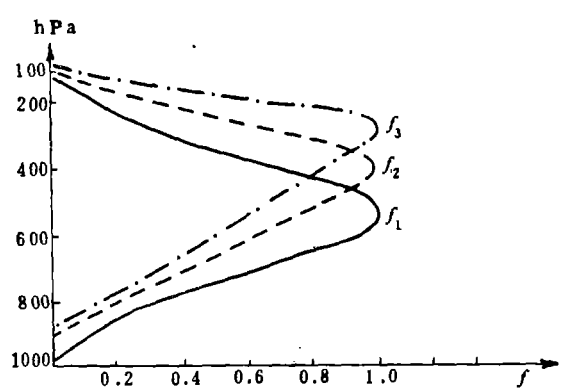

Fig. 6. Three types of profiles of $f(p)$.

Modifying $f_1(p)$ in the following way:

$$\begin{cases} f_2(p) = p/300 - 1/3, & \text{if } 100 \leqslant p \leqslant 400 \\ f_2(p) = -p/500 + 9/5, & \text{if } 400 < p \leqslant 900 \end{cases}$$

$$\begin{cases} f_3(p) = p/200 - 1/2, & \text{if } 100 < p \leqslant 300 \\ f_3(p) = -p/600 + 3/2, & \text{if } 300 < p \leqslant 900 \end{cases}$$

and prescribing $f(p) = 0$ at $p = 1000$ hPa, we can make $f_2(p)$ have the maximum at

400 hPa, and $f_3(p)$ have the maximum at 300 hPa. This distribution indicates that $f_3(p)$ represents the condition of deep convection, $f_1(p)$ relatively shallow convection and $f_2(p)$ some condition between the above two cases.

Fig. 7 shows M_C cross-sections for three different types of $f(p)$. The maximum of M_C is found at $r^*=0.5$ and its height varies for the different $f(p)$. When $f(p)$ is taken to be unity at 550 hPa, $M^*_{C\,\max}$ is also found at the same level. The same conditions are true for 400 hPa and 300 hPa. Thus, when $f(p)_{\max}$ goes upward, $M^*_{C\,\max}$ also moves up and at the same time the magnitudes of $M^*_{C\,\max}$ increase. If $f(p)=f_1(p), M^*_{C\,\max}=6.21$; if $f(p)=f_2(p), M^*_{C\,\max}=71.2$; if $f(p)=f_3(p), M^*_{C\,\max}=75.4$. These conditions are related to increase in depth of entrainment layer.

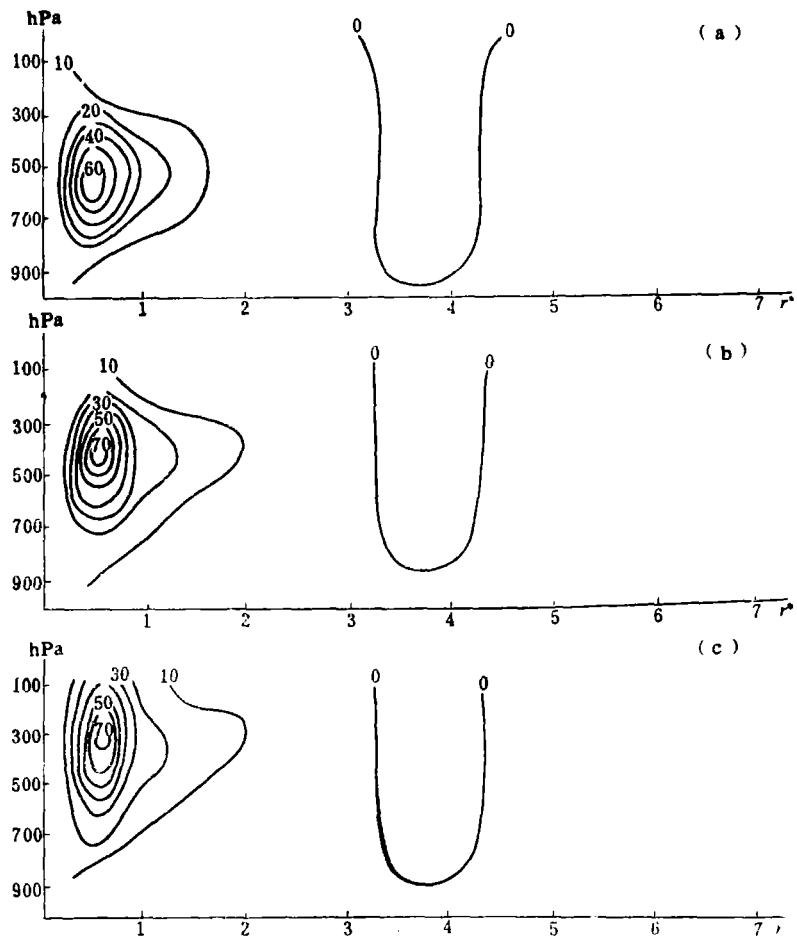

Fig. 7. M_C cross-sections for three different types of $f(p)$: (a) $f(p)=f_1(p)$; (b) $f(p)=f_2(p)$; ana (c) $f(p)=f_3(p)$ (units: hPa/s).

Fig. 8 shows the secondary circulations forced by Cu vertical flux of heat, with $f_2(p)$ and $f_3(p)$ taken for estimate of M_C. The center of the secondary circulation extends upward as the maximum of M_C has higher level, and the intensity of the circulation center also increases as M_C enhances. When $f(p)=f_1(p)$, the circulation center is located at about 500hPa, with the maximum being 30.3 (Fig. 4) while the circulation center moves

upward to 300hPa and the maximum is 53.0 when $f(p)=f_2(p)$ (Fig. 8a). When $f(p)=f_3(p)$, the circulation center is located just above 300 hPa and the maximum intensity is 80.5 (Fig. 8b).

Therefore, the adjustment of M_c may exert a significant effect on the height and intensity of the center of secondary circulation.

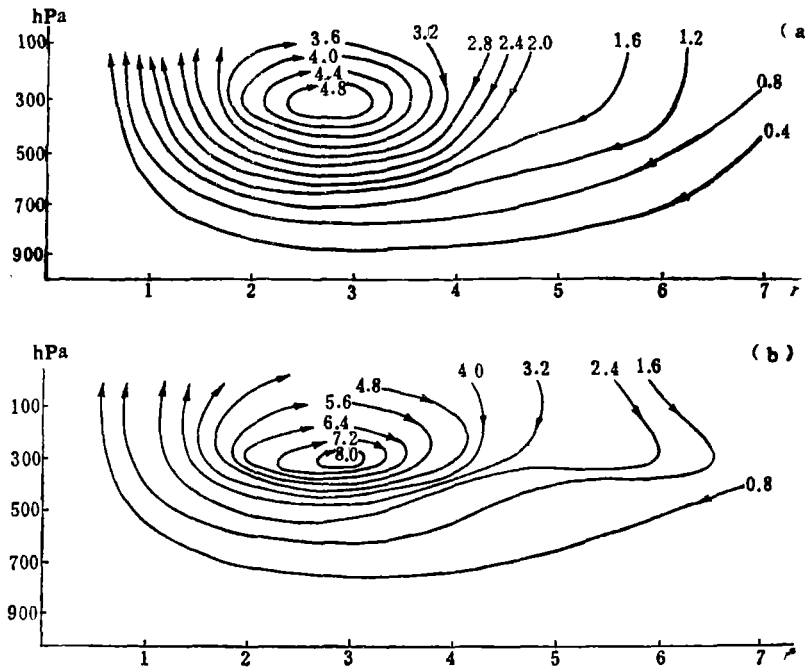

Fig. 8. The secondary circulation forced by Cu vertical mixing of heat for $f(p)=f_2(p)$ (a) and $f(p)=f_3(p)$ (b). The magnitude of order of ψ is 10^{-2}.

VI. EKMAN PUMPING AND Cu MIXING OF HEAT

In the process of typhoons, Ekman pumping plays an important role in the large-scale convergence. Thus, this term will be taken into account to examine its effect on the secondary circulation.

At the top of the boundary layer we have:

$$\omega_E=-\rho g(k/2f)^{1/2}\zeta_g, \quad \zeta_g=\frac{1}{r}\frac{\partial}{\partial r}(rv_g),$$

where ζ_g is the geostrophic vorticity and v_g the geostrophic wind. This term having been included, the secondary circulations forced by diabatic heating and Cu vertical mixing of heat tend to intensify at a certain degree. The enhancement of the former may be anticipated. We shall only discuss the latter.

Fig. 9 is the secondary circulation forced by Cu vertical mixing of heat with inclusion of Ekman pumping. As compared with Fig. 4, the forced circulation has a considerable degree of intensification. The physical process behind this enhancement may be described as follows: Ekman pumping effect can influence the vertical velocity at the top of boundary

layer and further the secondary circulation of typhoons. Once the secondary circulation gets intensified, it would in turn stimulate the pumping effect. As a result the above two processes work cooperatively to cause the rapid development of typhoons.

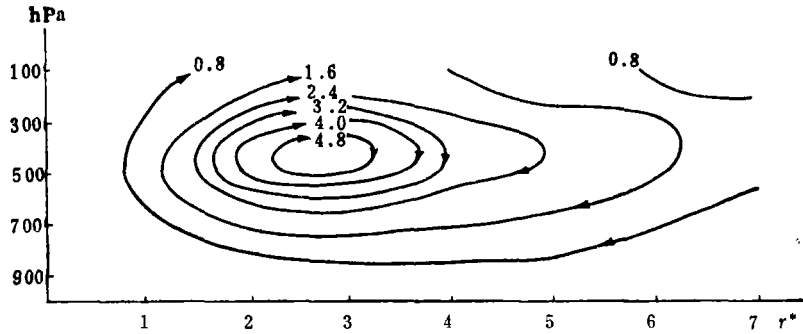

Fig. 9. The secondary circulation forced by Cu vertical mixing of heat with inclusion of Ekman pumping (magnitude of order of 10^{-2}).

VII. CONCLUSIONS

The present paper used a non-dimensional secondary circulation equation for typhoons and 11-yr compositing typhoon data to estimate the thermally forced secondary circulation. The main results have been obtained as follows:

(1) The effect of turbulent horizontal flux and Cu horizontal mixing of heat are of minor importance which may be neglected.

(2) The turbulent vertical flux of heat may force a positive circulation in the lower and middle troposphere with the magnitude of order of ψ being 10^{-4}.

(3) The diabatic heating and Cu vertical mixing of heat are major thermal forcing factors. They both may force the vigorous secondary circulation with ψ being 10^{-2}.

(4) The intensity of secondary circulation is rather sensitive to Cu mass flux M_c.

(5) Ekman pumping and Cu vertical heat mixing cooperatively work. This feedback process is favorable for the enhancement of the secondary circulation of typhoons.

REFERENCES

Ding Yihui and Liu Yuezhen (1985), A study of budget of kinetic energy in typhoon, *Scientia Sinica, Series B*, No. 11, 1045—1054 (in Chinese).

Eliassen, A. (1951), Slow thermally or frictionally controlled meridional circulations in a circular vortex, Astrophys. Norv., **5**: 19—60.

Eliassen, A. (1962), On the vertical circulation in frontal zones, *Geofys. Publ. Geophys. Norv.*, **24**: 147—160.

Krishnamurti, T. N. et al. (1984), Cumulus parameterization and rainfall rates, I: *Mon Wea. Rev.*, **108**: 465—477.

Kuo, H. L. (1974), Further studies of the parameterization of influence of cumulus convection on large scale flow. *J. Atmos. Sci.*, **31**: 1232—1240.

Lindzen, R. S. (1980), Report of workshop held at NASA Goddard Institute for Space Studies, New York, October 29—31, pp.42—51.

Sawyer, J. S. (1956), The vertical circulation at meteorological fronts and its relation to frontogenesis, *Proc. Roy. Soc.*, London, **A234**: 346—362.

Schneider, E. K. and Lindzen, R. S. (1976), A discussion of the parameterization of momentum exchange by cumulus convection, *J. Geophys. Res.*, **81**: 3158—3160.

Shapiro, M. A. (1981), Frontogenesis and geostrophically forced secondary circulation in the vicinity of jet-stream frontal zone systems, *J. Atmos. Sci.*, **38**: 954—973.

Shapiro, L. J. and Willoughby, H. E. (1982), The responses of balanced hurricanes to local sources of heat and momentum, *J. Atmos. Sci.*, **39**: 378—394.

Sun Zipin and Ding Yihui (1989), Effect of the dynamic forcing on the secondary circulation of typhoons, *Scientia Atmospherica Sinica*, 13.

Willoughby, H. E. (1979), Forced secondary circulation in hurricanes, *J. Geophys. Res.*, **84**: 3173—3183.