$$g_{21}=(1+m_2^2)(\lambda+Q\mu_2)+2\mu \tag{30f}$$

It is shown from the expressions of the displacement components that for the supersonic case there are three plane shock waves attached to the load and propagating with the velocities $1/\lambda_1$, $1/\lambda_2$ and $1/\lambda_3$ of the dilatational and rotational waves, respectively, as shown in Fig. 5.

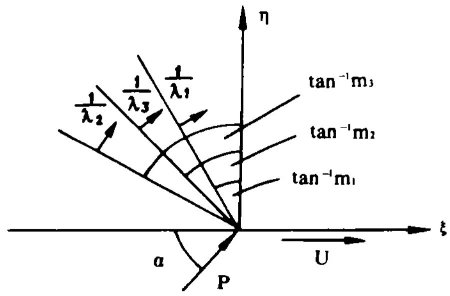

Fig. 5 Supersonic sketch.

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无限饱水孔隙弹性空间中由运动源所产生的位移场*

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摘要

基于 Biot 的饱和孔隙弹性介质的运动方程和利用复变函数方法，本文研究了无限饱水孔隙弹性空间中由常速运动源所产生的位移场。考虑了两种类型源：a. 沿无限空间水平轴运动的斜向集中力源；b. 运动双力偶源。关于力源的运动速度，考虑了4种情形：a. 力源的运动速度 U 小于饱水孔隙弹性介质的三种体波速度——亚音速情形；b. 速度 U 小于介质的第一纵波速度和横波速度，但大于第二纵波速度——弱跨音速情形；c. 速度 U 小于第一纵波速度，但大于横波速度和第二纵波速度——强跨音速情形；d. 速度 U 大于介质的所有三种体波速度——超音速情形。结果表明，在跨音速和超音速情形里，解呈现出与力源相联系的平面冲击波特征，位移出现了相应的跳跃。

主题词： 双力偶源 弹性介质 地震波速度 位错 位移场 常速运动源

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DISPLACEMENT FIELD DUE TO MOVING SOURCE IN AN INFINITE SATURATED POROUS ELASTIC SPACE*

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Abstract

Based on Biot' s basic motion equations for the fluid saturated porous elastic medium and by using the method of complex function, the displacement field due to moving source with uniform velocity U in an infinite porous elastic saturated space is studied in present paper. Two types of sources are considered: a. An oblique concentrated pulse force moving along the horizontal axis of the infinite space. b. The moving couples of forces. There are four cases for the moving velocity of force source: a. The velocity U is less than all three types of body wave velocities of the fluid saturated porous elastic medium, the subsonic case. b. The velocity U is less than the first dilatational wave velocity and the rotational wave velocity of the medium, but greater than the second dilatational wave velocity, the weak transonic case. c. The velocity U is less than the first dilatational wave velocity, but greater than the velocities of the second dilatational and the rotational waves, the strong transonic case. d. The velocity U is greater than all three types of body wave velocities, the supersonic case. The results show that in the transonic and supersonic cases, the solutions represent the character of plane shock waves attached to the load and associated with a jump in the displacement.

Key words: Double-couple source, Elastic medium, Seismic wave velocity, Dislocation, Displacement field, Moving source with uniform velocity

1 Introduction

In the past years, the problem of displacement in an elastic space caused by an impulse of pressure moving with constant velocity along a straight line has been studied by some authors. For example, I. N. Sneddon[1] and J. M. R. Radok[2] used the method of complex function to study the case that an oblique concentrated line force moves on the boundary of an elastic half space with constant velocity. Foregoing authors only considered the case of subsonic movement about the impulse of pressure. A. C. Eringen and E. S. Suhubi[3] studied the same problem in more detail and expanded the subsonic case into the transonic and supersonic cases about the movement velocity of the pulse force.

The problem of force source plays an important part in seismology. In the previous papers [4]、[5], the authors studied the problem of displacement and stress fields caused by a transient point force in an infinite porous elastic saturated space. By using the Laplace and Fourier transform methods, the solutions of the problem were analytically obtained for the δ-pulse point force and the Heaviside step function form of force. In the present paper, based on Biot' s basic motion equations for the fluid saturated porous elastic medium and also by means

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of the method of complex function, first of all, the authors discuss the problem of a concentrated pulse force moving with constant velocity in a saturated infinite elastic space. Then, we obtain the solutions for the moving double couples by differentiating the results with respect to the spatial coordinate variables. In these two cases, we have the solutions for a pulse force moving with subsonic, transonic and supersonic velocities.

2 Motion Equations and Boundary Conditions

For simplification and as a beginning, in the present paper we neglect the friction effect at the contact surface between solid phase and liquid phase, namely the dispersivity of the elastic waves is neglected. In this assumption, according to Biot' s theory, the problem of plane elasto-dynamics for fluid saturated porous medium is reduced to solve the following motion equations for four potential functions φ,ψ,H,K:

$$P\nabla^2\varphi+Q\nabla^2\psi=\frac{\partial^2}{\partial t^2}(\rho_{11}\varphi+\rho_{12}\psi) \tag{1a}$$

$$Q\nabla^2\varphi+R\nabla^2\psi=\frac{\partial^2}{\partial t^2}(\rho_{12}\varphi+\rho_{22}\psi) \tag{1b}$$

where, $P=\lambda+2\mu$, λ and μ are, respectively, the Lame' constants of the solid phase materials, Q is of the nature of a coupling between the volume change of the solid phase and that of the fluid phase, and it is of the dimention of stress, the coefficient R is a measure of the pressure required on the fluid to force a certain volume of the fluid into the pore while the total volume remains constant. ρ_{11} and ρ_{22} are respectively the total effective mass of the solid and that of the fluid in the relative motion between the solid and the fluid, ρ_{12} represents a mass coupling parameter between solid and fluid, it is of the dimention of mass and shows that when the solid is accelerated a force must be exerted on the fluid to prevent an average displacement of the latter. The preceding parameters must be satisfied with the following inequalities:

$$\rho_{11}>0,\ \rho_{22}>0,\ \rho_{12}<0 \tag{2a}$$

$$\rho_{11}\rho_{22}-\rho_{12}^2>0,\ PR-Q^2>0 \tag{2b}$$

By means of the preceding four potential functions, the displacements $\vec{u}$, $\vec{U}$ for the solid and the fluid, the stress components σ_{ij} acting on the solid, and the porous fluid pressure σ can be represented as:

$$u_x=\frac{\partial\varphi}{\partial x}-\frac{\partial H}{\partial y},\ U_x=\frac{\partial\psi}{\partial x}-\frac{\partial K}{\partial y} \tag{3a}$$

$$u_y=\frac{\partial\varphi}{\partial y}+\frac{\partial H}{\partial x},\ U_y=\frac{\partial\psi}{\partial y}+\frac{\partial K}{\partial x} \tag{3b}$$

$$\sigma_{xx}=P\nabla^2\varphi+Q\nabla^2\psi-2\mu(\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2 H}{\partial x\partial y}) \tag{4a}$$

$$\sigma_{yy}=P\nabla^2\varphi+Q\nabla^2\psi-2\mu(\frac{\partial^2\varphi}{\partial x^2}-\frac{\partial^2 H}{\partial x\partial y}) \tag{4b}$$

$$\sigma_{xy}=\mu(2\frac{\partial^2\varphi}{\partial x\partial y}+\frac{\partial^2 H}{\partial x^2}-\frac{\partial^2 H}{\partial y^2}) \tag{4c}$$

$$\sigma=Q\nabla^2\varphi+R\nabla^2\psi \tag{4d}$$

If we introduce the complex variable z and its conjugate $\bar{z}$:

$$z=x+iy,\ \bar{z}=x-iy$$

and notice the relations

$$2\frac{\partial}{\partial z}=\frac{\partial}{\partial x}-i\frac{\partial}{\partial y},\ 2\frac{\partial}{\partial \bar{z}}=\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}$$

the displacements and the stresses can be rewritten in the complex form as following:

$$D=u_x+iu_y=2\frac{\partial}{\partial \bar{z}}(\varphi+iH) \tag{5a}$$

$$\Theta=\sigma_{xx}+\sigma_{yy}=8(\lambda+\mu)\frac{\partial^2\varphi}{\partial z\partial \bar{z}}+8Q\frac{\partial^2\psi}{\partial z\partial \bar{z}} \tag{5b}$$

$$\Phi=\sigma_{xx}-\sigma_{yy}+2i\sigma_{xy}=4\mu\frac{\partial D}{\partial \bar{z}} \tag{5c}$$

$$\sigma=4Q\frac{\partial^2\varphi}{\partial z\partial \bar{z}}+4R\frac{\partial^2\psi}{\partial z\partial \bar{z}} \tag{5d}$$

Let

$$\varphi=\varphi_1+\varphi_2,\ \psi=\mu_1\varphi_1+\mu_2\varphi_2,\ K=\gamma_1 H \tag{6}$$

and substituting them into equations (1), one has

$$(\nabla^2-\lambda_{1,2}^2\frac{\partial^2}{\partial t^2})\varphi_{1,2}=0 \tag{7a}$$

$$(\nabla^2-\lambda_3^2\frac{\partial^2}{\partial t^2})H=0 \tag{7b}$$

where

$$\lambda_{1,2}^2=\frac{b\pm\sqrt{b^2-4ac}}{2a},\ \lambda_3^2=\frac{C}{\mu\rho_{22}} \tag{8a}$$

$$\mu_{1,2}=\frac{R\rho_{11}-Q\rho_{12}-a\lambda_{1,2}^2}{Q\rho_{22}-R\rho_{12}},\ \gamma_1=\frac{\rho_{12}}{\rho_{22}} \tag{8b}$$

$$a=PR-Q^2,\ b=R\rho_{11}+P\rho_{22}-2Q\rho_{12},\ c=\rho_{11}\rho_{22}-\rho_{12}^2 \tag{8c}$$

Now, assuming the disturbed source moves with the constant velocity U along the axis x in an infinite fluid porous elastic space, and let

$$\xi=x-Ut,\ \eta=y$$

the equation (7) becomes

$$(1-M_{1,2}^2)\frac{\partial^2\varphi_{1,2}}{\partial\xi^2}+\frac{\partial^2\varphi_{1,2}}{\partial\eta^2}=0,\ (1-M_3^2)\frac{\partial^2 H}{\partial\xi^2}+\frac{\partial^2 H}{\partial\eta^2}=0 \tag{9}$$

Above, $M_j=U\lambda_j, j=1,2,3$ are the Mach numbers of the moving source relative to the dilatational and rotational waves, respectively. If $M_j<1$, the corresponding equation is elliptic, and it is of the solution of form $\omega_j(z_j)+\overline{\omega_j(z_j)}$, in which z_j is a complex variable. If $M_j>1$, the equation is hyperbolic, and has the solution of form $\omega_j^+(\xi+m_j\eta)+\omega_j^-(\xi-m_j\eta)$. As figure 1 shows, the boundary conditions are

$$u_x^+(\xi,0)=u_x^-(\xi,0),\ U_x^+(\xi,0)=U_x^-(\xi,0) \tag{10a}$$

$$u_y^+(\xi,0)=u_y^-(\xi,0),\ U_x^+(\xi,0)=U_x^-(\xi,0) \tag{10b}$$

$$\sigma_{yy}^+(\xi,0)+\sigma_{yy}^-(\xi,0)=-P\sin\alpha\ \delta(\xi) \tag{10c}$$

$$\sigma_{xy}^+(\xi,0)+\sigma_{xy}^-(\xi,0)=-P\cos\alpha\ \delta(\xi) \tag{10d}$$

in which $u_x^+(\xi, 0)$, $u_x^-(\xi, 0)$, et al. represent the value of the functions $u_x^-(\xi, \eta)$, $u_x^+(\xi,\eta)$ et al. on the real axis as approached from the positive and negative half-space, respectively. In the following sections, we shall solve the problem under the boundary conditions (10) for various values of velocity U.

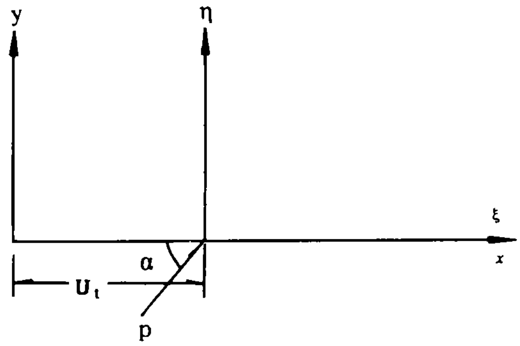

Fig. 1 Moving force and coordinate systems.

3 Subsonic Case

What is called as the subsonic case is that the velocity of the moving force source is less than all three kinds of body wave velocities of the fluid saturated porous elastic medium, namely, $M_1<1$, $M_2<1$, $M_3<1$. In this case, all three equations of (9) are elliptic, and thier solutions can be written as

$$\varphi_1(z_1)=\omega_1(z_1)+\overline{\omega_1(z_1)},\quad \varphi_2(z_2)=\omega_2(z_2)+\overline{\omega_2(z_2)},\quad H=i[\omega_3(z_3)-\overline{\omega_3(z_3)}] \tag{11}$$

where

$$z_j=\xi+i\beta_j\eta,\quad \beta_j=(1-M_j^2)^{\frac{1}{2}},\quad j=1,2,3 \tag{12}$$

Substituting them into equations (3), (4) and the boundary conditions (10), one has ω_j, $j=1,2,3$. Finally, we obtain the displacement and stress fields in infinite fluid saturated porous elastic space for the subsonic case. For instance, in the positive half-space, one has the displacement field as following:

$$u_x^+=\frac{P}{2\pi}[(\gamma_1-\mu_2)(\frac{\beta_3\log r_1\cos\alpha}{\mu A}+\frac{\beta_2\theta_1\sin\alpha}{B})+(\mu_1-\gamma_1)(\frac{\beta_3\log r_2\cos\alpha}{\mu A}+\frac{\beta_1\theta_2\sin\alpha}{B})-$$
$$-\beta_3(\mu_1-\mu_2)(\frac{\log r_3\cos\alpha}{\mu A}+\frac{\beta_1\beta_2\theta_3\sin\alpha}{B})] \tag{13a}$$

$$u_y^+=-\frac{P}{2\pi}[\beta_1(\gamma_1-\mu_2)(\frac{\beta_3\theta_1\cos\alpha}{\mu A}-\frac{\beta_2\log r_1\sin\alpha}{B})+\beta_2(\mu_1-\gamma_1)(\frac{\beta_3\theta_2\cos\alpha}{\mu A}-\frac{\beta_1\log r_2\sin\alpha}{B})-$$
$$-(\mu_1-\mu_2)(\frac{\theta_3\cos\alpha}{\mu A}-\frac{\beta_1\beta_2\log r_3\sin\alpha}{B})] \tag{13b}$$

where

$$A=(\gamma_1-\mu_2)(1+\beta_3^2-2\beta_1\beta_3)+(\mu_1-\gamma_1)(1+\beta_3^2-2\beta_2\beta_3) \tag{14a}$$
$$B=\beta_2(\gamma_1-\mu_2)(f_1+2\beta_1\beta_3\mu)+\beta_1(\mu_1-\gamma_1)(f_2+2\beta_2\beta_3\mu) \tag{14b}$$
$$f_1=(1-\beta_1^2)(\lambda+Q\mu_1)-2\beta_1^2\mu \tag{14c}$$
$$f_2=(1-\beta_2^2)(\lambda+Q\mu_2)-2\beta_2^2\mu \tag{14d}$$

By differentiating the preceding results with respect to the spatial variables, one has the solutions for the moving double couples, as shown in Figure 2:

$$u_x^+=\frac{M_0}{2\pi}[\frac{(\gamma_1-\mu_2)}{r_1^2}(\frac{\beta_3(\xi+\beta_1^2\eta)\cos\alpha}{\mu A}+\frac{\beta_1\beta_2(\xi-\eta)\sin\alpha}{B})+\frac{(\mu_1-\gamma_1)}{r_2^2}(\frac{\beta_3(\xi+\beta_2^2\eta)\cos\alpha}{\mu A}+$$
$$+\frac{\beta_1\beta_2(\xi-\eta)\sin\alpha}{B})-\frac{\beta_3(\mu_1-\mu_2)}{r_3^2}(\frac{(\xi+\beta_3^2\eta)\cos\alpha}{\mu A}+\frac{\beta_1\beta_2\beta_3(\xi-\eta)\sin\alpha}{B})] \tag{15a}$$

$$u_y^+=-\frac{M_0}{2\pi}[\frac{\beta_1(\gamma_1-\mu_2)}{r_1^2}(\frac{\beta_1\beta_3(\xi-\eta)\cos\alpha}{\mu A}-\frac{\beta_2(\xi+\beta_1^2\eta)\sin\alpha}{B})+\frac{\beta_2(\mu_1-\gamma_1)}{r_2^2}(\frac{\beta_2\beta_3(\xi-\eta)\cos\alpha}{\mu A}-$$

$$-\frac{\beta_1(\xi+\beta_2^2\eta)sin\alpha}{B})-\frac{(\mu_1-\mu_2)}{r_3^2}(\frac{\beta_3(\xi-\eta)cos\alpha}{\mu A}-\frac{\beta_1\beta_2(\xi+\beta_3^2\eta)sin\alpha}{B})] \qquad (15b)$$

Above, M_0 is the moment of the couple at the couple source, and

$$r_j=(\xi^2+\beta_j^2\eta^2)^{\frac{1}{2}},\ \theta_j=tan^{-1}\frac{\beta_j\eta}{\xi},\ j=1,2,3 \qquad (16)$$

4 Weak Transonic Case

In the fluid saturated porous elastic space, there are three elastic body waves: two dilatational waves, one rotational wave. Among these waves, the velocity of the first dilatational wave is fastest and the velocity of the second dilatational wave is slowest, but the damping corresponding to the second dilatational wave is greatest. The transonic case indicates that the moving velocity of the disturbed source in the elastic space is greater than one or two among the three elastic wave velocities. If the moving velocity of the disturbed source is only greater than the velocity of the second dilatational wave, the effect on the disturbed field is not great, it is called as "weak transonic case". In this case, $M_1<1, M_2>1, M_3<1$, and the equations associated with M_1, M_3 are elliptic, the rest is hyperbolic, Let the solutions of these three equations are: in the positive half-space

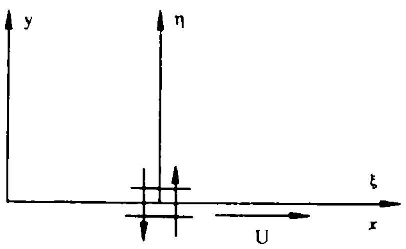

Fig. 2 Moving double couples.

$$\varphi_1^+(z_1)=\omega_1^+(z_1)+\overline{\omega_1^+(z_1)},\ \varphi_2^+(\xi+m_2\eta)=2\omega_2^+(\xi+m_2\eta),$$
$$H^+(z_3)=i[\omega_3^+(z_3)-\overline{\omega_3^+(z_3)}\ ] \qquad (17)$$
$$m_2=(M_2^2-1)^{\frac{1}{2}}>0$$

in the negative half-space

$$\varphi_1^-(z_1)=\omega_1^-(z_1)+\overline{\omega_1^-(z_1)},\ \varphi_2^-(\xi-m_2\eta)=2\omega_2^-(\xi-m_2\eta),$$
$$H^-(z_3)=i[\omega_3^-(z_3)-\overline{\omega_3^-(z_3)}\ ] \qquad (18)$$

By means of the equations (3), (4) and the boundary conditions (10), one has all these potential functions $\varphi_1^+, \varphi_1^-, \varphi_2^+, \varphi_2^-, H^+, H^-$, and then we obtain the displacement field (in the positive half-space) as following:

$$u_x^+=\frac{P}{2\pi}\{(\gamma_1-\mu_2)[\frac{\beta_3(G_2\log r_1+G_1\theta_1)cos\alpha}{\mu(G_1^2+G_2^2)}-\frac{(R_1\log r_1-R_2\theta_1)sin\alpha}{R_1^2+R_2^2}]-\beta_3(\mu_1-\mu_2)\cdot$$
$$\cdot[\frac{(G_2\log r_3+G_1\theta_3)cos\alpha}{\mu(G_1^2+G_2^2)}-\frac{\beta_1(R_1\log r_3-R_2\theta_3)sin\alpha}{R_1^2+R_2^2}]\}+\frac{(\mu_1-\gamma_1)P}{2}\{\frac{\beta_3cos\alpha}{\mu(G_1^2+G_2^2)}\cdot$$
$$\cdot[\frac{G_2}{\pi}\log(\xi+m_2\eta)-G_1H(\xi+m_2\eta)]+\frac{\beta_1sin\alpha}{m_2(R_1^2+R_2^2)}[\frac{R_2}{\pi}\log(\xi+m_2\eta)-$$
$$-R_1H(\xi+m_2\eta)]\} \qquad (19a)$$

$$u_y^+=-\frac{P}{2\pi}\{\beta_1(\gamma_1-\mu_2)[\frac{\beta_3(G_2\theta_1-G_1\log r_1)cos\alpha}{\mu(G_1^2+G_2^2)}-\frac{(R_1\theta_1+R_2\log r_1)sin\alpha}{R_1^2+R_2^2}]-(\mu_1-\mu_2)\cdot$$
$$\cdot[\frac{(G_2\theta_3-G_1\log r_3)cos\alpha}{\mu(G_1^2+G_2^2)}-\frac{\beta_1(R_1\theta_3+R_2\log r_3)sin\alpha}{R_1^2+R_2^2}]\}+\frac{m_2(\mu_1-\gamma_1)P}{2}\{\frac{\beta_3cos\alpha}{\mu(G_1^2+G_2^2)}\cdot$$
$$\cdot[\frac{G_2}{\pi}\log(\xi+m_2\eta)-G_1H(\xi+m_2\eta)]+\frac{\beta_1sin\alpha}{m_2(R_1^2+R_2^2)}[\frac{R_2}{\pi}\log(\xi+m_2\eta)-$$

$$-R_1H(\xi+m_2\eta)]\} \tag{19b}$$

where, H(t) is the Heaviside step function, and

$$G_1=2m_2\beta_3(\mu_1-\gamma_1) \tag{20a}$$

$$G_2=(1+\beta_3^2)(\mu_1-\mu_2)-2\beta_1\beta_3(\gamma_1-\mu_2) \tag{20b}$$

$$R_1=g_{22}\beta_1(\mu_1-\gamma_1)/m_2 \tag{20c}$$

$$R_2=f_1(\gamma_1-\mu_2)+2\beta_1\beta_3\mu(\mu_1-\mu_2) \tag{20d}$$

$$g_{22}=(1+m_2^2)(\lambda+Q\mu_2)+2\mu m_2^2 \tag{20e}$$

For the moving double couples we obtain (in the positive half-space):

$$u_x^+=\frac{M_0}{2\pi}\{\frac{(\gamma_1-\mu_2)}{r_1^2}[\frac{\beta_3(G_2(\xi+\beta_1^2\eta)+G_1\beta_1(\xi-\eta))cos\alpha}{\mu(G_1^2+G_2^2)}-\frac{(R_1(\xi+\beta_1^2\eta)-R_2\beta_1(\xi-\eta))sin\alpha}{R_1^2+R_2^2}-$$
$$-\frac{\beta_3(\mu_1-\mu_2)}{r_3^2}[\frac{(G_2(\xi+\beta_3^2\eta)+G_1\beta_3(\xi-\eta)cos\alpha}{\mu(G_1^2+G_2^2)}-\frac{\beta_1(R_1(\xi+\beta_3^2\eta)-R_2\beta_3(\xi-\eta)sin\alpha}{R_1^2+R_2^2}]\}+$$
$$+\frac{(\mu_1-\mu_2)(1+m_2)M_0}{2}\{\frac{\beta_3cos\alpha}{\mu(G_1^2+G_2^2)}[\frac{G_2}{\pi(\xi+m_2\eta)}-G_1\delta(\xi+m_2\eta)]+\frac{\beta_1sin\alpha}{m_2(R_1^2+R_2^2)}\cdot$$
$$\cdot[\frac{R_2}{\pi(\xi+m_2\eta)}-R_1\delta(\xi+m_2\eta)]\} \tag{21a}$$

$$u_y^+=-\frac{M_0}{2\pi}\{\frac{\beta_1(\gamma_1-\mu_2)}{r_1^2}[\frac{\beta_3(G_2\beta_1(\xi-\eta)-G_1(\xi+\beta_1^2\eta))cos\alpha}{\mu(G_1^2+G_2^2)}-$$
$$-\frac{(R_1\beta_1(\xi-\eta)+R_2(\xi+\beta_1^2\eta))sin\alpha}{R_1^2+R_2^2}-\frac{(\mu_1-\mu_2)}{r_3^2}\cdot$$
$$\cdot[\frac{(G_2\beta_3(\xi-\eta)-G_1(\xi+\beta_3^2\eta))cos\alpha}{\mu(G_1^2+G_2^2)}-\frac{\beta_1(R_1\beta_3(\xi-\eta)+R_2(\xi+\beta_3^2\eta))sin\alpha}{R_1^2+R_2^2}]\}+$$
$$+\frac{m_2(\mu_1-\gamma_1)(1+m_2)M_0}{2}\{\frac{\beta_3cos\alpha}{\mu(G_1^2+G_2^2)}[\frac{G_2}{\pi(\xi+m_2\eta)}-G_1\delta(\xi+m_2\eta)]+\frac{\beta_1sin\alpha}{m_2(R_1^2+R_2^2)}\cdot$$
$$\cdot[\frac{R_2}{\pi(\xi+m_2\eta)}-R_1\delta(\xi+m_2\eta)]\} \tag{21b}$$

where, $\delta(t)$ is the Dirac pulse function.

It is shown from the preceding results that the parts in the displacements caused by the function ω_2 are the functions of argument $\xi+m_2\eta=x+m_2y-Ut$, and are with the plane wave character. Furthermore, the expressions such as $\delta(x+m_2y-Ut)$ and $H(x+m_2y-Ut)$ appear in the displacement components, which represent a plane shock wave $x+m_2y-Ut=0$ attached to the load and associated with an impulse and a jump in the displacement components (Fig. 3). In the actual situation, the plane shock wave due to the velocity of the second dilatational wave being less than the moving velocity of the disturbed source will be dispered rapidly.

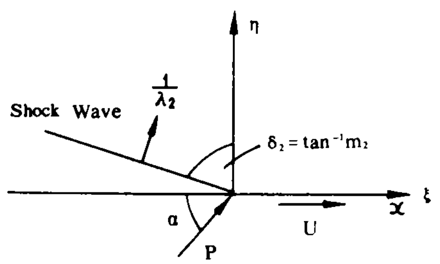

Fig. 3 Weak transonic sketch.

5 Strong Transonic Case

In this case, there are $M_1<1, M_2>1, M_3>1$, and the equation associated with M_1 is elliptic, the rests are hyperbolic. Let the solutions of these equations (7) are : in the positive half-space

$$\varphi_1^+(z_1)=\omega_1^+(z_1)+\overline{\omega_1^+(z_1)},\ \varphi_2^+(\xi+m_2\eta)=2\omega_2^+(\xi+m_2\eta),$$
$$H^+(\xi+m_3\eta)=2\omega_3^+(\xi+m_2\eta) \tag{22}$$
$$m_2=(M_2^2-1)^{\frac{1}{2}},m_3=(M_3^2-1)^{\frac{1}{2}}$$

in the negative half-space

$$\varphi_1^-(z_1)=\omega_1^-(z_1)+\overline{\omega_1^-(z_1)},\ \varphi_2^-(\xi-m_2\eta)=2\omega_2^-(\xi-m_2\eta),$$
$$H^-(\xi-m_3\eta)=2\omega_3^-(\xi-m_3\eta) \tag{23}$$

For the displacement field in the positive half-space, one has:

$$u_x^+=-\frac{P}{2\pi}\left[\frac{(2\beta_1\log r_1+\theta_1G_3)cos\alpha}{\mu(G_3^2+4\beta_1^2)}-\frac{(R_3\log r_1+\theta_1f_1)sin\alpha}{f_1^2+R_3^2}\right]-\frac{(\mu_1-\gamma_1)P}{2(\gamma_1-\mu_2)}\cdot$$
$$\cdot\left\{\frac{cos\alpha}{\mu(G_3^2+4\beta_1^2)}\left[\frac{2\beta_1\log(\xi+m_2\eta)}{\pi}-G_3H(\xi+m_2\eta)\right]-\frac{\beta_1sin\alpha}{m_2(f_1^2+R_3^2)}\left[\frac{f_1\log(\xi+m_2\eta)}{\pi}+\right.\right.$$
$$\left.\left.+R_3H(\xi+m_2\eta)\right]\right\}+\frac{(\mu_1-\mu_2)P}{2(\gamma_1-\mu_2)}\left\{\frac{cos\alpha}{\mu(G_3^2+4\beta_1^2)}\left[\frac{2\beta_1\log(\xi+m_3\eta)}{\pi}-G_3H(\xi+m_3\eta)\right]+\right.$$
$$\left.+\frac{m_3\beta_1sin\alpha}{f_1^2+R_3^2}\left[\frac{f_1\log(\xi+m_3\eta)}{\pi}+R_3H(\xi+m_3\eta)\right]\right\} \tag{24a}$$

$$u_y^+=-\frac{P}{2\pi}\left[\frac{(G_3\log r_1-2\theta_1\beta_1)cos\alpha}{\mu(G_3^2+4\beta_1^2)}-\frac{(f_1\log r_1-\theta_1R_3)sin\alpha}{f_1^2+R_3^2}\right]-\frac{(\mu_1-\gamma_1)P}{2(\gamma_1-\mu_2)}\cdot$$
$$\cdot\left\{\frac{m_2cos\alpha}{\mu(G_3^2+4\beta_1^2)}\left[\frac{2\beta_1\log(\xi+m_2\eta)}{\pi}-G_3H(\xi+m_2\eta)\right]-\frac{\beta_1 sin\alpha}{f_1^2+R_3^2}\left[\frac{f_1\log(\xi+m_2\eta)}{\pi}+\right.\right.$$
$$\left.\left.+R_3H(\xi+m_2\eta)\right]\right\}-\frac{(\mu_1-\mu_2)P}{2(\gamma_1-\mu_2)}\left\{\frac{cos\alpha}{\mu m_3(G_3^2+4\beta_1^2)}\left[\frac{2\beta_1\log(\xi+m_3\eta)}{\pi}-G_3H(\xi+m_3\eta)\right]+\right.$$
$$\left.+\frac{\beta_1sin\alpha}{f_1+R_3^2}\left[\frac{f_1\log(\xi+m_3\eta)}{\pi}+R_3H(\xi+m_3\eta)\right]\right\} \tag{24b}$$

For the displacement field due to the moving double couples, one has (in the positive half-space):

$$u_x^+=-\frac{M_0}{2\pi r_1^2}\left[\frac{\beta_1(2(\xi+\beta_1^2\eta)+G_3(\xi-\eta))cos\alpha}{\mu(G_3^2+4\beta_1^2)}-\frac{(f_1\beta_1(\xi-\eta)+R_3(\xi+\beta_1^2\eta))sin\alpha}{f_1^2+R_3^2}\right]-$$
$$-\frac{(\mu_1-\gamma_1)(1+m_2)M_0}{2(\gamma_1-\mu_2)}\left\{\frac{cos\alpha}{\mu(G_3^2+4\beta_1^2)}\left[\frac{2\beta_1}{\pi(\xi+m_2\eta)}-G_3\delta(\xi+m_2\eta)\right]-\frac{\beta_1sin\alpha}{m_2(f_1^2+R_3^2)}\cdot\right.$$
$$\left.\cdot\left[\frac{f_1}{\pi(\xi+m_2\eta)}+R_3\delta(\xi+m_2\eta)\right]\right\}+\frac{(\mu_1-\mu_2)(1+m_3)M_0}{2(\gamma_1-\mu_2)}\left\{\frac{cos\alpha}{\mu(G_3^2+4\beta_1^2)}\left[\frac{2\beta_1}{\pi(\xi+m_3\eta)}-\right.\right.$$
$$\left.\left.-G_3\delta(\xi+m_3\eta)\right]+\frac{m_3\beta_1sin\alpha}{f_1^2+R_3^2}\left[\frac{f_1}{\pi(\xi+m_3\eta)}+R_3\delta(\xi+m_3\eta)\right]\right\} \tag{25a}$$

$$u_y^+=-\frac{M_0}{2\pi r_1^2}\left[\frac{(2\beta_1^2(\xi-\eta)-G_3(\xi+\beta_1^2\eta))cos\alpha}{\mu(G_3^2+4\beta_1^2)}+\frac{(f_1(\xi+\beta_1^2\eta)-R_3\beta_1(\xi-\eta))sin\alpha}{f_1^2+R_3^2}\right]-$$
$$-\frac{(\mu_1-\gamma_2)(1+m_2)M_0}{2(\gamma_1-\mu_2)}\left\{\frac{m_2cos\alpha}{\mu(G_3^2+4\beta_1^2)}\left[\frac{2\beta_1}{\pi(\xi+m_2\eta)}-G_3\delta(\xi+m_2\eta)\right]-\frac{\beta_1sin\alpha}{f_1^2+R_3^2}\cdot\right.$$
$$\left.\cdot\left[\frac{f_1}{\pi(\xi+m_2\eta)}+R_3\delta(\xi+m_2\eta)\right]\right\}+\frac{(\mu_1-\mu_2)(1+m_3)M_0}{2(\gamma_1-\mu_2)}\left\{\frac{cos\alpha}{\mu m_3(G_3^2+4\beta_1^2)}\left[\frac{2\beta_1}{\pi(\xi+m_3\eta)}-\right.\right.$$
$$\left.\left.-G_3\delta(\xi+m_3\eta)\right]+\frac{\beta_1sin\alpha}{f_1^2+R_3^2}\left[\frac{f_1}{\pi(\xi+m_3\eta)}+R_3\delta(\xi+m_3\eta)\right]\right\} \tag{25b}$$

It is shown from the expressions for the displacement components that in the strong transonic case there are two shock waves $x+m_2y-Ut=0$ and $x+m_3y-Ut=0$ (as shown in

Figure 4).

6 Supersonic Case

In this case the moving velocity U of the disturbed source exceeds all three velocites of the body waves, namely, one has $M_1>1, M_2>1, M_3>1$, and all three equations associated with them are hyperbolic. Let the solutions of these equations are: in the positive half-space

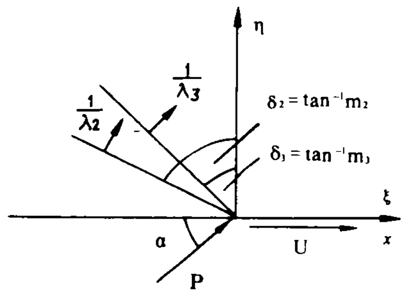

Fig. 4 Strong transonic sketch.

$$\varphi_1^+(\xi+m_1\eta)=2\omega_1^+(\xi+m_1\eta),$$
$$\varphi_2^+(\xi+m_2\eta)=2\omega_2^+(\xi+m_2\eta),$$
$$H^+(\xi+m_3\eta)=2\omega_3^+(\xi+m_3\eta) \tag{26}$$
$$m_1=(M_1^2-1)^{\frac{1}{2}}, m_2=(M_2^2-1)^{\frac{1}{2}}, m_3=(M_3^2-1)^{\frac{1}{2}}$$

in the negative half-space

$$\varphi_1^-(\xi-m_1\eta)=2\omega_1^-(\xi-m_1\eta),\ \varphi_2^-(\xi-m_2\eta)=2\omega_2^-(\xi-m_2\eta),$$
$$H^-(\xi-m_3\eta)=2\omega_3^-(\xi-m_3\eta) \tag{27}$$

The displacement field generated by the preceding potential functions is (in the positive half-space):

$$u_x^+=-\frac{P}{2}[(\gamma_1-\mu_2)(\frac{m_2 sin\alpha}{R_4}+\frac{m_3 cos\alpha}{\mu G_4})H(\xi+m_1\eta)+(\mu_1-\gamma_1)(\frac{m_1 sin\alpha}{R_4}+\frac{m_3 cos\alpha}{\mu G_4})\cdot$$
$$\cdot H(\xi+m_2\eta)-m_3(\mu_1-\mu_2)(\frac{m_1 m_2 sin\alpha}{R_4}+\frac{cos\alpha}{\mu G_4})H(\xi+m_3\eta)] \tag{28a}$$
$$u_y^+=-\frac{P}{2}[m_1(\gamma_1-\mu_2)(\frac{m_2 sin\alpha}{R_4}+\frac{m_3 cos\alpha}{\mu G_4})H(\xi+m_1\eta)+(\mu_1-\gamma_1)(\frac{m_1 sin\alpha}{R_4}+\frac{m_3 cos\alpha}{\mu G_4})\cdot$$
$$\cdot H(\xi+m_2\eta)+(\mu_1-\mu_2)(\frac{m_1 m_2 sin\alpha}{R_4}+\frac{cos\alpha}{\mu G_4})H(\xi+m_3\eta)] \tag{28b}$$

For the displacement field caused by the moving double couples, one obtains

$$u_x^+=-\frac{M_0}{2}[(1+m_1)(\gamma_1-\mu_2)(\frac{m_2 sin\alpha}{R_4}+\frac{m_3 cos\alpha}{\mu G_4})\delta(\xi+m_1\eta)+(1+m_2)(\mu_1-\gamma_1)\cdot$$
$$\cdot(\frac{m_1 sin\alpha}{R_4}+\frac{m_3 cos\alpha}{\mu G_4})\delta(\xi+m_2\eta)-m_3(1+m_3)(\mu_1-\mu_2)(\frac{m_1 m_2 sin\alpha}{R_4}+\frac{cos\alpha}{\mu G_4})\delta(\xi+$$
$$+m_3\eta)] \tag{29a}$$
$$u_y^+=-\frac{M_0}{2}[m_1(1+m_1)(\gamma_1-\mu_2)(\frac{m_2 sin\alpha}{R_4}+\frac{m_3 cos\alpha}{\mu G_4})\delta(\xi+m_1\eta)+m_2(1+m_2)(\mu_1-\gamma_1)\cdot$$
$$\cdot(\frac{m_1 sin\alpha}{R_4}+\frac{m_3 cos\alpha}{\mu G_4})\delta(\xi+m_2\eta)+(1+m_3)(\mu_1-\mu_2)(\frac{m_1 m_2 sin\alpha}{R_4}+\frac{cos\alpha}{\mu G_4})\delta(\xi+m_3\eta)] \tag{29b}$$

Above,

$$G_3=\frac{(\mu_1-\mu_2)(m_3^2-1)-2m_2 m_3(\mu_1-\gamma_1)}{m_3(\gamma_1-\mu_2)} \tag{30a}$$
$$R_3=\frac{2m_3\mu\beta_1(\mu_1-\mu_2)-g_{22}\beta_1(\mu_1-\gamma_1)}{m_2(\gamma_1-\mu_2)} \tag{30b}$$
$$G_4=(\gamma_1-\mu_2)(2m_1 m_3-m_3^2+1)+(\mu_1-\gamma_1)(2m_2 m_3-m_3^2+1) \tag{30c}$$
$$R_4=m_2(\gamma_1-\mu_2)(g_{11}-2\mu m_1 m_3)+m_1(\mu_1-\gamma_1)(g_{21}-2\mu m_2 m_3) \tag{30d}$$
$$g_{11}=(\lambda+\mu Q)(1+m_1^2)+2\mu \tag{30e}$$

$$g_{21}=(1+m_2^2)(\lambda+Q\mu_2)+2\mu \tag{30f}$$

It is shown from the expressions of the displacement components that for the supersonic case there are three plane shock waves attached to the load and propagating with the velocities $1/\lambda_1$, $1/\lambda_2$ and $1/\lambda_3$ of the dilatational and rotational waves, respectively, as shown in Fig. 5.

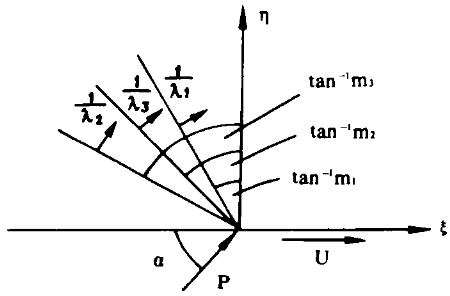

Fig. 5 Supersonic sketch.

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无限饱水孔隙弹性空间中由运动源所产生的位移场*

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摘要

基于Biot的饱和孔隙弹性介质的运动方程和利用复变函数方法，本文研究了无限饱水孔隙弹性空间中由常速运动源所产生的位移场。考虑了两种类型源：a. 沿无限空间水平轴运动的斜向集中力源；b. 运动双力偶源。关于力源的运动速度，考虑了4种情形：a. 力源的运动速度U小于饱水孔隙弹性介质的三种体波速度——亚音速情形；b. 速度U小于介质的第一纵波速度和横波速度，但大于第二纵波速度——弱跨音速情形；c. 速度U小于第一纵波速度，但大于横波速度和第二纵波速度——强跨音速情形；d. 速度U大于介质的所有三种体波速度——超音速情形。结果表明，在跨音速和超音速情形里，解呈现出与力源相联系的平面冲击波特征，位移出现了相应的跳跃。

主题词： **双力偶源**　**弹性介质**　**地震波速度**　**位错**　位移场　常速运动源

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