ELEMENTARY VIEW OF GREENHOUSE EFFECT OF THE ATMOSPHERES OF VARIOUS PLANETS *

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ABSTRACT

The global mean temperatures of the atmosphere and the surface of various planets of the solar system are determined by taking the system as in radiative equilibrium, with the atmosphere taken as transparent to solar radiation but with an albedo α, and composed of N layers each of which absorbs all infrared radiation that falls on it, and a top layer of partial absorptivity a, while the surface is taken as black. It is found that, for the earth's atmosphere with $\alpha=0.33$, $N=0$, $a=0.83$, it gives the current observed mean surface temperature $T_s=15°C$ and the effective mean radiative temperature of the atmosphere $T_a=242.6K$. On the other hand; the atmosphere of Venus is characterized by $\alpha=0.70$ and $N=70$, which yields a surface temperature of about 700K.

It is also found that surface evaporation and absorption of solar radiation by the atmosphere tend to lower the surface temperature.

Key words: radiation balance, greenhouse effect, global mean temperature, atmospheres of various planets

I. PURE RADIATIVE BALANCE

First we take the global mean temperature of the atmosphere and the surface of the planet as in pure radiative balance by considering the radiative process in terms of the spectral mean absorptivity of the atmosphere for infrared radiation and taking it as transparent for solar radiation, except contributing to a part of the effective albedo α. The surface is taken as radiating as a black body. In order to cover the case of the very thick atmosphere of Venus and that of the earth and other planets, we take the atmosphere as composed of N layers, each of which is just able to absorb all the infrared radiation which falls on it, that is, as N stacked black bodies with regard to infrared radiation, which we shall number downward, and a top layer with partial absorptivity a whose value is between zero and one. The global mean of solar radiation received by a unit area of the surface per unit time is

$$S=(1-\alpha)\,I_0\,/\,4 \quad , \tag{1}$$

where I_0 is the solar intensity at the surface level and α is the albedo. Radiation balance then requires that the outgoing radiation through the top of the atmosphere be equal to S. The part of the outgoing radiation given by the top layer is $a\sigma T_a^4$, and the second part is from the black layer below it, and this amount is $(1-a)\sigma T_1^4$, where T_a is the temperature of the top partially transparent layer in degree K, and σ is the Stefan–Boltzmann constant. When $N=0$, T_1 is then equal to the surface temperature T_s. Thus we have

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* The paper is written to the memory of Dr. Tu Changwang (1906—1962), one of the founders for modern meteorology of China.

$$aT_a^4 + (1-a)\,T_1^4 = S/\sigma \ . \tag{2}$$

Since the top layer radiates through both its top and bottom surfaces but receives radiation only from the layer below, we have

$$2\,T_a^4 = T_1^4 \ . \tag{3}$$

From these two relations we find

$$T_a^4 = \frac{S/\sigma}{2-a} \ , \tag{4a}$$

$$T_1^4 = \frac{2\,S/\sigma}{2-a} \ . \tag{4b}$$

Since all the layers below are black, their radiative balance condition is given by

$$2\,T_n^4 = T_{n-1}^4 + T_{n+1}^4 \ . \tag{5}$$

Thus, by working with increasing n we find we have the following general formula

$$T_{n+1}^4 = \frac{2(n+1)-na}{2-a} \times \frac{S}{\sigma} \ , \tag{6}$$

which holds for all n in the range $0<n<N$, and with $T_s = T_{N+1}$.

The earth's atmosphere is semi–transparent, which corresponds to $N=0$; $0<a<1$. Further, we have $I_0 = 1380\mathrm{W/m^2}$, and $\alpha = 0.33$, hence $S/\sigma = 0.4237 \times 10^{10}\mathrm{K}^4$. The mean surface temperature T_s and the mean radiative temperature T_a corresponding to this value of S/σ and five different values of a are given in Table 1. The current mean surface temperature is 15°C, which corresponds to $a=0.83$, according to (6).

Table 1. T_s and T_a Corresponding to $S/\sigma = 0.4237 \times 10^{10}\mathrm{K}^4$

a	0	0.25	0.50	0.75	0.83	1.00
T_a (K)	212.0	219.4	228.0	238.7	242.6	252.3
T_s (K)	252.4	260.9	271.1	283.8	288.5	300.1

On the other hand, the atmosphere of Venus is about a hundred times heavier than earth's atmosphere and most of it is $CO_2$and hence it can be taken as consisting of a large number of completely absorbing layers. Observations show that the albedo of Venus is about 0.70, so that the value of S/σ for Venus is $1.1676 \times 10^{10}\mathrm{K}^4$. Thus, with $N=50$ we find its radiative surface temperature is 650 K, while for $N=70$, T_s will be 706 K.

II. INFLUENCE OF EVAPORATION EFFECT

The upward heat flux across the earth's surface is not only through radiation, but also by evaporation from the water surface, that is, through latent heat flux. For simplicity, we set the latent heat flux as equal to b times σT_s^4. Then the thermal balance condition for the earth is

$$(1+b)\,T_s^4 = \frac{S}{\sigma} + a\,T_a^4 \ . \tag{7}$$

The balance condition at the top of the atmosphere is still given by

$$(1-a)\,T_s^4 + a\,T_a^4 = \frac{S}{\sigma}\ . \tag{8}$$

On the other hand, the thermal balance condition for the atmosphere is now given by

$$2\,T_a^4 = (1+\frac{b}{a})\,T_0^4\ . \tag{9}$$

From these relations we find we have

$$T_s^4 = \frac{2}{2-a+b} \times \frac{S}{\sigma}\ , \tag{10a}$$

$$T_a^4 = \frac{1+b/a}{2-a+b} \times \frac{S}{\sigma}\ . \tag{10b}$$

These results indicate that the influence of evaporation from the water surface is to reduce T_s and to increase T_a, and therefore, it is a negative feedback for the surface temperature, even though it has also increased the downward radiation by the atmosphere. Since evaporation rate increases rapidly with increasing surface temperature, this negative feedback may play a very significant role for the thermal balance at the surface. The influences of water vapor and clouds on solar radiation must also be included.

III. INFLUENCE OF SOLAR RADIATION ABSORPTION BY THE ATMOSPHERIC GASES ON GREENHOUSE EFFECT

The solar radiation is also absorbed by the water vapor, CO_2 and O_3 in the atmosphere, and this effect must be taken into consideration in determining the greenhouse effect of the atmosphere on the surface temperature. For simplicity, we represent this effect by the mean absorptivity a_s of the atmosphere for solar radiation, so that the amount of solar energy absorbed by the atmosphere is $a_s S$ while $(1-a_s)\,S$ is absorbed by the surface. The condition at the top of the atmosphere under radiative balance is still

$$a\,T_a^4 + (1-a)\,T_s^4 = S/\sigma\ . \tag{11}$$

However, under the circumstance the energy balance for the atmosphere as a whole is given by

$$2\,T_a^4 = \left(1+\frac{b}{a}\right)T_s^4 + \frac{a_s}{a} \times \frac{S}{\sigma}\ . \tag{12}$$

Consequently we have

$$T_a^4 = \frac{1}{2-a+b} \times \left[1 + \frac{b-(1-a)a_s}{a}\right] \times \frac{S}{\sigma}\ , \tag{13a}$$

$$T_s^4 = \frac{1}{2-a+b} \times \left[2 - \left(1+\frac{b}{a}\right)a_s\right] \times \frac{S}{\sigma}\ . \tag{13b}$$

These results indicate that both b and a_s make T_a increase and T_s decrease. These solutions also satisfy the surface balance condition

$$(1+b)\,T_s^4 = (1-a_s)\frac{S}{\sigma} + aT_a^4\ .$$