RESPONSE OF TROPICAL ATMOSPHERIC CIRCULATION TO THE OCEAN-LAND SURFACE HEATING —AN ANALYTICAL SIMULATION

Ji Zhengang (季振刚) and *Chao Jiping* (巢纪平)

National Research Center for Marine Environment Forecasts, State Oceanic Administration, Beijing

Received April 1, 1988

ABSTRACT

A linear steady model is constructed to investigate the response of the tropical atmosphere to diabatic heating. The basic equations are similar to those used by Gill (1980), but the long-wave approximation is removed and periodic boundary conditions are taken in longitude. According to the features of the underlying surface temperature (including oceans and land), the heat sources (sinks) are given. Using this analytical model, we have simulated the climatological fields of wind and air pressure in the lower layers of the tropical and subtropical atmosphere in summer (June—August) and winter (December—February). The main features of observations are depicted in simulated fields.

I. INTRODUCTION

The research of tropical dynamics have made great advances since the pioneering work of Bjerknes (1966). Webster (1972) drew attention to the response of the tropical atmosphere to the steady and local forcing by using a numerical model. Gill (1980) constructed a simple analytical model to elucidate some basic features of the response of the tropical atmosphere to diabatic heating. He clarified the different functions of Kelvin wave and Rossby wave in tropical circulations. Heckley and Gill (1984) extended Gill's work to time-dependent problems. However, in their works the long-wave approximation and infinite boundary condition were still used, thus resulting in inconsistency with observations. Gill and Philips (1986, 1987) developed Gill's work (1980) and discussed the influences of nonlinear effects and basic wind fields with analytical models. Nevertheless, they did not take the periodic boundary condition of the real atmosphere and did not attempt to simulate the climatological fields of the tropical atmosphere.

Since the pioneering calculations of Philips (1956), Smagorinsky (1963) and Mintz (1965), the numerical models of the global atmosphere have been made much progress. The climatological fields of the global atmosphere have been simulated by a number of general circulation models. Gates et al. (1985) set up a coupled air-sea interaction model and simulated the global circulation in January successfully.

The simulations mentioned above are all numerical investigations. There are few authors using analytical models to simulate the climatological fields of the atmosphere until the present time. In this paper we will construct a linear steady analytical model to simulate the climatological fields of the tropical and subtropical atmosphere. In the model we will remove long-wave approximation and use periodic boundary conditions in longitude.

II. THE MODEL

1. *Basic Equations*

To research heat forcing atmospheric motions, the following basic equations are often used:

$$\partial u/\partial t - yv/2 = -\partial p/\partial x, \tag{1}$$
$$\partial v/\partial t + yu/2 = -\partial p/\partial y, \tag{2}$$
$$\partial p/\partial t + \partial u/\partial x + \partial v/\partial y = -Q, \tag{3}$$
$$w = \partial p/\partial t + Q. \tag{4}$$

For the sake of obtaining steady response of the atmosphere to heat forcing, some dissipative processes should be included. The commonly used and convenient ones are Newtonian cooling and Rayleigh friction, which lead the atmosphere to a steady state. For mathematical simplicity, Newtonian cooling and Rayleigh friction are assumed to have same dissipative coefficient ε. Thus Eqs. (1)—(4) are reduced to

$$\varepsilon u - yv/2 = -\partial p/\partial x, \tag{5}$$
$$\varepsilon v + yu/2 = -\partial p/\partial y, \tag{6}$$
$$\varepsilon p + \partial u/\partial x + \partial v/\partial y = -Q, \tag{7}$$
$$w = \varepsilon p + Q. \tag{8}$$

Non-dimensional scale $L_a = (C/2\beta)^{1/2}$ is the so-called Rossby deformation radius of the tropical atmosphere, in which

$$C = (gH)^{1/2} = ND/\pi, \tag{9}$$

where all the signs have common meteorological meaning. We take $H = 400$ m, $C = 60$ m s^{-1} and $L_a = 1000$ km. For the sake of mathematical simplicity, the zonal mean wind is not taken into account in our model. The influences of this approximation upon the simulations will be discussed later. To the present stage, our model is similar to Gill's (1980). Matsuno (1966), Zebiak (1982) and many other authors discussed the physical properties of Eqs. (1)—(4) quite clearly. Here we will not explain Eqs. (1)—(8) in detail.

It is noticed that an assumption used in Gill's model (1980) is the long-wave approximation, i.e., $2\varepsilon k \ll 1$, where $k = \pi/2L$ is the wave number of heat source along the longitude, L represents the half width of heat source. Gill took $\varepsilon = 0.1$, Zebiak (1982) took $\varepsilon = 0.3$. If taking $\varepsilon = 0.2$ and $L = 1500$ km, we have $2\varepsilon k = 0.41$, which is not much less than 1. Therefore the long-wave approximation is valid only for the case of small ε and large-scale heat source. But the real distributions of heat sources in the tropical regions hardly satisfy those conditions, we will not take long-wave approximation in our model.

2. *Boundary Conditions*

As we know that the real atmosphere is circular around the earth, it is difficult to describe the response of the whole tropical atmosphere to heat source by use of infinite boundary conditions. In our model the periodic boundary conditions are used in longitude. The variables of u, v, w and p satisfy the following equations:

$$u(x+2\pi R, y) = u(x, y), \tag{10}$$
$$v(x+2\pi R, y) = v(x, y), \tag{11}$$
$$w(x+2\pi R, y) = w(x, y), \tag{12}$$
$$p(x+2\pi R, y) = p(x, y), \tag{13}$$

where R is the radius of the earth.

3. *Heat Sources*

Parabolic cylinder function $D_n(y)$ (cf. Abramowitz and Stegun, 1965) is used to solve Eqs. (5)—(8). The variables in Eqs. (5)—(8) are expanded in terms of $D_n(y)$ in the y-direction. For heat source $Q(x, y)$, we have

$$Q(x,y)=\sum_{n=0}^{\infty} F_n(x)D_n(y), \tag{14}$$

where distribution function $F_n(x)$ is described in six sections respectively, i.e.,

$$F_n(x)=\begin{cases} A_{m1}\cdot\cos k_1(x+M_1), & -(M_1+L_{11})\leqslant x\leqslant -(M_1-L_{12}) \\ a_{12}\cdot e^{-b_{12}(x+M_1-L_{12})}+a_{21}\cdot e^{b_{21}(x+L_{21})}, & -(M_1-L_{12})\leqslant x< -L_{21} \\ A_{m2}\cdot\cos k_2 x, & -L_{21}\leqslant x<L_{22} \\ a_{22}\cdot e^{-b_{22}(x-L_{22})}+a_{31}\cdot e^{b_{31}(x-M_3+L_{31})}, & L_{22}\leqslant x<M_3-L_{31} \\ A_{m3}\cdot\cos k_3 x(x-M_3), & M_3-L_{31}\leqslant x<M_3+L_{32} \\ a_{32}\cdot e^{-b_{32}(x-M_3-L_{32})}+a_{11}\cdot e^{b_{11}(x-M_3+L_{31})}, & M_3+L_{32}\leqslant x<M_2+M_3-L_{11} \end{cases} \tag{15}$$

where

$$a=A_{mi}\cdot\cos(k_i\cdot L_{ij}) \tag{16}$$

$$b=k_i\cdot \mathrm{tg}(k_i\cdot L_{ij}). \qquad i,j=1,2,3 \tag{17}$$

From the above expressions it is easy to know that three isolated heat sources or sinks are given by cosine functions. Their amplitudes are A_{m1}, A_{m2} and A_{m3} respectively. M_i ($i=$ 1, 2, 3) are the distance between two sources. The sum of M_1, M_2 and M_3 is the perimeter of the earth, namely,

$$M_1+M_2+M_3=2\pi R. \tag{18}$$

In order to get continuous solutions of Eqs. (5)—(8), $Q(x, y)$ and dQ/dx should be continuous. In Eq. (15) three exponential functions are used to make $F_n(x)$ and $dF_n(x)/dx$ continuous. It should be pointed out that the heat source $Q(x, y)$ given by Gill (1980) can not make dF_n/dx continuous at some special points.

In the present model we assume that if temperature of the underlying surface (including oceans and land) is higher than circumstance temperature, there is a heat source in this area. Conversely, there is a heat sinks. The circumstance temperature is the annual and zonal averaged temperature in the lower atmosphere. The meanings of u, v, w and p are determined by forcing term $Q(x, y)$. As $Q(x, y)$ is proportional to the difference between underlying surface temperature and the annual and zonal averaged temperature in the lower atmosphere, u, v, w, and p should be proportional to the departures of the annual and zonal means of zonal wind, meridional wind, vertical velocity and sea level pressure respectively. Their mathematical expressions are

$$Q(x,y,t)=Q_0[T_1^*(x,y,t)-\int_0^T\int_0^{2\pi} T_1^*(x,y,t)\,dt\,dx], \tag{19}$$

$$u(x,y,t)=u_0[u^*(x,y,t)-\int_0^T\int_0^{2\pi} u^*(x,y,t)\,dt\,dx], \tag{20}$$

$$v(x,y,t)=v_0[v^*(x,y,t)-\int_0^T\int_0^{2\pi} v^*(x,y,t)\,dt\,dx], \tag{21}$$

$$p(x,y,t)=p_0[p^*(x,y,t)-\int_0^T\int_0^{2\pi} p^*(x,y,t)\,dt\,dx], \tag{22}$$

where T_1^*, u^*, v^* and p^* denote the climatological air temperature, zonal wind, meridional wind and sea level pressure. All of them can have seasonal variations. Parameters u_0, v_0, and p_0 are constants. T equals one year. The integrations in

Eqs.(19)—(22) represent annual and zonal means.

It should be pointed out that according to observations, the high (low) temperature areas are not the heat sources (sinks) in some cases. For example, water vapor may not release its latent heat just at the place where it is evaporated. The background fields of the atmosphere can influence precipitation significantly. But for a first preliminary approximation, we may believe that the assumption mentioned in the last paragraph is valid.

III. ANALYTICAL SOLUTIONS

In order to lead the reducing procedure to be more clear, only important results and formulas are shown. Some expressions of coefficients which are too complex will not be offered in this paper.

1. *Symmetric Forcing (I)*: $Q=F_0(x)D_0(y)$

In this case only the first term of the heating function in Eq. (14) is considered, other terms vanish. Eq. (14) is reduced to

$$Q(x,y)=F_0(x)D_0(y), \tag{23}$$

where distribution function $F_0(x)$ is determined by Eq. (15).

The three-dimensional velocities u,v,w and air pressure p can be expanded in terms of $D_n(y)$, i. e.,

$$u(x,y)=\sum_{n=0}^{\infty}U_n(x)D_n(y), \tag{24}$$

$$v(x,y)=\sum_{n=0}^{\infty}V_n(x)D_n(y), \tag{25}$$

$$p(x,y)=\sum_{n=0}^{\infty}P_n(x)D_n(y), \tag{26}$$

$$w(x,y)=\sum_{n=0}^{\infty}W_n(x)D_n(y). \tag{27}$$

For the sake of solving Eqs. (5)—(8) more efficiently, Eqs. (23)—(26) are put into Eqs. (5)—(7) directly. Using the normal property of $D_n(y)$ and the boundary conditions (10)—(13), we can have the following expressions:

$$u(x,y)=U_0(x)D_0(y)+U_2(x)D_2(y), \tag{28}$$

$$v(x,y)=V_1(x)D_1(y), \tag{29}$$

$$p(x,y)=P_0(x)D_0(y)+P_2(x)D_2(y), \tag{30}$$

and a set of equations

$$\varepsilon\frac{d^2V_1}{dx^2}+\frac{1}{2}\frac{dV_1}{dx}-\varepsilon(\varepsilon^2+\frac{3}{2})V_1=\frac{1}{2}[\varepsilon F_0(x)+F_0'(x)], \tag{31}$$

$$\frac{dU_0}{dx}+\varepsilon U_0=-\left[(\varepsilon^2+1)V_1+\varepsilon\frac{dV_1}{dx}+\frac{1}{2}F_0\right], \tag{32}$$

$$U_2=\frac{1}{8}\cdot\varepsilon(-2\varepsilon^2+1)V_1+\frac{1}{4}\cdot\frac{dV_1}{dx}-\frac{1}{8}\varepsilon\cdot F_0, \tag{33}$$

$$P_0=2\varepsilon V_1+U_0+4U_2 \tag{34}$$

$$P_2=U_2. \tag{35}$$

Thus solving partial differential Eqs. (5)—(8) is reduced to solving Eqs. (31)—(35). In fact, only the solutions of ordinary differential Eqs. (31) and (32) are needed, and thereby the complete solutions of Eqs. (5)—(8) can be obtained.

The solutions of Eqs. (31) and (32) are

$$V_1(x)=\begin{cases} H_1\cdot e^{K_1(x+M_1)}+H_2\cdot e^{-K_2(x+M_1)}+C_{11}\sin k_1(x+M_1)+C_{12}\cos k_1(x+M_1), \\ \qquad -(M_1+L_{11})\leqslant x<-(M_1-L_{12}) \\ H_3\cdot e^{K_1(x+M_1/2)}+H_4\cdot e^{-K_2(x+M_1/2)}+d_{12}\cdot e^{-b_{12}(x-M_1-L_{12})}+d_{21}\cdot e^{b_{21}(x+L_{21})}, \\ \qquad -(M_1-L_{12})\leqslant x<-L_{21} \\ H_5e^{K_1x}+H_6\cdot e^{-K_2x}+C_{21}\sin k_2x+C_{22}\cos k_2x, \quad -L_{21}\leqslant x<L_{22} \\ H_7\cdot e^{K_1(x-M_3/2)}+H_8\cdot e^{-K_2(x-M_3/2)}+d_{22}e^{-b_{22}(x-L_{22})}+d_{31}e^{b_{31}(x-M_3+L_{31})}, \\ \qquad L_{22}\leqslant x<M_3-L_{31} \\ H_9e^{K_1(x-M_3)}+H_{10}e^{-K_2(x-M_3)}+C_{31}\sin k_3(x-M_3)+C_{32}\cdot\cos k_3(x-M_3), \\ \qquad M_3-L_{31}\leqslant x<M_3+L_{32} \\ H_{11}e^{K_1(x-M_3-M_2/2)}+H_{12}e^{-K_2(x-M_3-M_2/2)}+d_{32}e^{-b_{32}(x-M_3-L_{32})}+d_{11}\cdot e^{(x-M_2-M_3+L_{11})}, \\ \qquad M_3+L_{32}\leqslant x<M_2+M_3-L_{11} \end{cases} \tag{36}$$

$$U_0(x)=\begin{cases} E_{11}e^{K_1(x+M_1)}+E_{12}e^{-K_2(x+M_1)}+E_{13}\sin k_1(x+M_1)+E_{14}\cos k_1(x+M_1) \\ \qquad +E_{15}e^{-\varepsilon(x+M_1)}, \quad -(M_1+L_{11})\leqslant x<-(M_1-L_{12}) \\ E_{21}e^{K_1(x+M_1/2)}+E_{22}\cdot e^{-K_2(x+M_1/2)}+E_{23}e^{-b_{12}(x+M_1-L_{12})}+E_{24}\cdot e^{b_{21}(x+L_{21})} \\ \qquad \cdot e^{-\varepsilon(x+M_1/2)}E_{25}\cdot e^{-\varepsilon(x+m_1/2)}, \quad -(M_1-L_{12})\leqslant x<-L_{21} \\ E_{31}e^{K_1x}+E_{32}e^{K_2x}+E_{33}\sin k_2x+E_{34}\cos k_2x+E_{35}e^{-\varepsilon x}, \\ \qquad -L_{21}\leqslant x<L_{22} \\ E_{41}e^{K_1(x-M_3/2)}+E_{42}e^{-K_2(x-M_3/2)}+E_{43}e^{-b_{22}(x-L_{22})}+E_{44}e^{b_{31}(x-M_3+L_{31})}+E_{45} \\ \qquad \cdot e^{-\varepsilon(x-M_3/2)}, \quad L_{22}\leqslant x<M_3-L_{31} \\ E_{51}e^{K_1(x-M_3)}+E_{52}e^{-K_2(x-M_3)}+E_{53}\sin k_3(x-M_3)+E_{54}\cos k_3(x-M_3) \\ \qquad +E_{55}\cdot e^{-\varepsilon(x-M_3)}, \quad M_3-L_{31}\leqslant x<M_3+L_{32} \\ E_{61}e^{K_1(x-M_3-M_2/2)}+E_{62}\cdot e^{-K_2(x-M_3-M_2/2)}+E_{63}e^{-b_{32}(x-M_3-L_{32})}+E_{64}e^{b_{11}(x-M_2-M_3+L_{11})} \\ \qquad +E_{65}e^{-\varepsilon(x+M_3-M_2/2)}, \quad M_3+L_{32}\leqslant x\leqslant M_2+M_3-L_{11} \end{cases} \tag{37}$$

2. *Asymmetric Forcing:* $Q=F_1(x)D_1(y)$

This is the case that the unique non-zero term in Eq. (14) corresponds to $n=1$, which is given by

$$Q(x,y)=F_1(x)D_1(y), \tag{38}$$

where the form of $F_1(x)$ is determined by Eq. (15).

The same technique can be applied to this case. Substituting Eqs. (24)—(26) and (38) into Eqs. (5)—(7), we can obtain

$$u(x,y)=U_1(x)D_1(y)+U_3(x)D_3(y), \tag{39}$$
$$v(x,y)=V_0(x)D_0(y)+V_2(x)D_2(y), \tag{40}$$
$$p(x,y)=P_1(x)D_1(y)+P_3(x)D_3(y), \tag{41}$$

and

$$2\varepsilon\frac{dU_0}{dx}+(2\varepsilon^2+1)U_0=F_1(x)\,, \tag{42}$$

$$4\varepsilon(d^2U_3/dx^2)+2(dU_3/dx)-(4\varepsilon^3+10\varepsilon)U_3=F_1(x), \tag{43}$$

$$V_2(x)=2\varepsilon U_3+2(dU_3/dx), \tag{44}$$

$$U_1(x)=-[2\varepsilon(dU_3/dx)+(3+2\varepsilon^2)U_3+\varepsilon V_0], \tag{45}$$

$$P_1(x)=-2\varepsilon V_1-U_1, \tag{46}$$

$$P_3(x)=U_3. \tag{47}$$

The solutions of Eqs. (42) and (43) are

$$V_0(x)=\begin{cases} G_{11}\sin k_1(x+M_1)+G_{12}\cos k_1(x+M_1)+G_{13}e^{-(\varepsilon+1/2\varepsilon)(x+M_1)}, \\ \qquad -(M_1+L_{11})\leqslant x<-(M_1+L_{12}) \\ G_{21}e^{-b_{12}(x+M_1-L_{12})}+G_{22}e^{b_{21}(x+L_{22})}+G_{23}e^{-(\varepsilon+1/2\varepsilon)(x+M_1/2)}, \\ \qquad -(M_1-L_{12})\leqslant x<-L_{21} \\ G_{31}\sin k_2x+G_{32}\cos k_2x+G_{33}e^{-(\varepsilon+1/2\varepsilon)x}, \qquad -L_{21}\leqslant x<L_{22} \\ G_{11}e^{-b_{22}(x-L_{22})}+G_{42}e^{b_{31}(x-M_2+L_{31})}+G_{43}e^{-(\varepsilon+1/2\varepsilon)(x-M_2/2)}, \\ \qquad L_{22}\leqslant x<M_3-L_{31} \\ G_{51}\sin k_3(x-M_3)+G_{52}\cos k_3(x-M_3)+G_{53}e^{-(\varepsilon+1/2\varepsilon)(x-M_3)}, \\ \qquad M_3-L_{31}\leqslant x<M_3+L_{32} \\ G_{61}e^{-b_{32}(x-M_2-L_{32})}+G_{62}e^{b_{11}(x-M_2-M_3+L_{11})}+G_{63}e^{-(\varepsilon+1/2\varepsilon)(x-M_3-M_2/2)}, \\ \qquad M_3+L_{32}\leqslant x<M_2+M_3-L_{11} \end{cases} \tag{48}$$

$$U_3(x)=\begin{cases} P_{11}e^{K_1(x+M_1)}+P_{12}e^{-K_2(x+M_1)}+P_{13}\sin k_1(x+M_1)+P_{14}\cos k_1(x+M_1), \\ \qquad -(M_1+L_{11})\leqslant x<-(M_1-L_{12}) \\ P_{21}e^{K_1(x+M_1/2)}+P_{22}e^{-K_2(x+M_1/2)}+P_{23}e^{-b_{12}(x+M_1-L_{12})}+P_{24}e^{b_{21}(x+L_{21})} \\ \qquad -(M_1-L_{12})\leqslant x<-L_{21} \\ P_{31}e^{K_1x}+P_{32}e^{-K_2x}+P_{33}\sin k_2x+P_{34}\cos k_2x, \quad -L_{21}\leqslant x<L_{22} \\ P_{41}e^{K_1(x-M_3/2)}+P_{42}e^{-K_2(x-M_3/2)}+P_{43}e^{-b_{22}(x-L_{22})}+P_{44}e^{b_{31}(x-M_3+L_{31})}, \\ \qquad L_{22}\leqslant x<M_3-L_{31} \\ P_{51}e^{K_1(x-M_3)}+P_{52}e^{-K_2(x-M_3)}+P_{53}\sin k_3(x-M_3)+P_{54}\cos k_3(x-M_3), \\ \qquad M_3-L_{31}\leqslant x<M_3+L_{32} \\ P_{61}e^{K_1(x-M_3-M_2/2)}+P_{62}e^{-K_2(x-M_3-M_2/2)}+P_{63}e^{-b_{32}(x-M_3-L_{32})} \\ \qquad +P_{64}e^{b_{11}(x-M_2-M_3+L_{11})}. \qquad M_3+L_{32}\leqslant x<M_2+M_3-L_{11} \end{cases} \tag{49}$$

Thus the complete solutions of partial differential Eqs. (5)—(8), which satisfy the periodic boundary conditions, are obtained as $Q(x,y)=F_1(x)D_1(y)$.

3. *Symmetric Forcing (II)*: $Q=F_2(x)D_2(y)$

In this case the unique non-zero term in Eq. (14) corresponds to $n=2$, which is given by

$$Q(x,y)=F_2(x)D_2(y)\,. \tag{50}$$

The expression of $F_2(x)$ is determined by Eq. (15). In the same procedures as in the above two cases, we finally obtain the following expressions:

$$u(x,y)=U_0(x)D_0(y)+U_2(x)D_2(y)+U_4(x)D_4(y), \tag{51}$$

$$v(x,y)=V_1(x)D_1(y)+V_3(x)D_3(y), \tag{52}$$

$$p(x,y)=P_0(x)D_0(y)+P_2(x)D_2(y)+P_4(x)D_4(y), \tag{53}$$

and

$$\frac{d^2U_4}{dx^2}+\frac{1}{2\varepsilon}\ \frac{dU_4}{dx}-\left(\varepsilon^2+\frac{7}{2}\right)U_4=\frac{1}{4\varepsilon}F_2, \tag{54}$$

$$\frac{d^2U_2}{dx^2}+\frac{1}{2\varepsilon}\ \frac{dU_2}{dx}-\left(\varepsilon^2+\frac{3}{2}\right)U_2=-\Big[4(\varepsilon^2+1)U_4$$

$$+4\varepsilon\frac{dU_4}{dx}+\frac{3}{4\varepsilon}F_2+\frac{dF_2}{dx}\Big], \tag{55}$$

$$\frac{dU_0}{dx}+\varepsilon U_0=-\Big\{[16\varepsilon^4+56\varepsilon^2+27]\varepsilon\cdot U_4+(16\varepsilon^4+24\varepsilon^2+3)\ \frac{dU_4}{dx}$$

$$+(4\varepsilon^2+5)\varepsilon U_2+(4\varepsilon^2+1)\ \frac{dU_2}{dx}+(4\varepsilon^2+1)F_2\Big\}, \tag{56}$$

$$P_0(x)=2\varepsilon V_1+4\varepsilon V_3+U_0+4U_2+16U_4, \tag{57}$$

$$P_2(x)=2\varepsilon V_3+U_2+8U_4, \tag{58}$$

$$P_4(x)=U_4, \tag{59}$$

$$V_3(x)=2\varepsilon U_4+2(dU_4/dx), \tag{60}$$

$$V_1(x)=4\varepsilon(2\varepsilon^2+11/2)U_4+4(2\varepsilon^2+3/2)\ \frac{dU_4}{dx}+2\varepsilon U_2$$

$$+2\frac{dU_2}{dx}+2F_2. \tag{61}$$

In a comparison of Eq. (43) with Eq. (54), it can be seen that the forms of these two ordinary differential equations are much the same. Therefore the solution of Eq. (54) should be similar to form (49).

$$U_2(x)=\begin{cases} R_{11}e^{BK_1(x+M_1)}+R_{12}e^{-BK_2(x+M_1)}+R_{13}e^{K_1(x+M_1)}+R_{14}e^{-K_2(x+M_1)}+R_{15}\sin k_1(x+M_1) \\ \quad +R_{16}\cos k_1(x+M_1), \qquad -(M_1+L_{11})\leqslant x<-(M_1-L_{12}) \\ R_{21}e^{BK_1(x+M_1/2)}+R_{22}e^{-BK_2(x+M_1/2)}+R_{23}e^{K_1(x+M_1/2)}+R_{24}e^{-K_2(x+M_1/2)} \\ \quad +R_{25}e^{-b_{12}(x+M_1-L_{12})}+R_{26}e^{b_{21}(x+L_{21})}, \qquad -(M_1-L_{12})\leqslant x<-L_{21} \\ R_{31}e^{BK_1x}+R_{32}e^{-BK_2x}+R_{33}e^{K_1x}+R_{34}e^{-K_2x}+R_{35}\sin k_2x+R_{36}\cos k_2x, \\ \quad -L_{21}\leqslant x<L_{22} \\ R_{41}e^{BK_1(x-M_3/2)}+R_{42}e^{-BK_2(x-M_3/2)}+R_{43}e^{K_1(x-M_3/2)}+R_{44}e^{-K_2(x-M_3/2)} \\ \quad +R_{45}e^{-b_{22}(x-L_{22})}+R_{46}e^{b_{31}(x-M_3+L_{31})}, \qquad L_{22}\leqslant x<M_3-L_{31} \\ R_{51}e^{BK_1(x-M_3)}+R_{52}e^{-BK(x-M_3)}+R_{53}e^{K_1(x-M_3)}+R_{54}e^{-K_2(x-M_3)} \\ \quad +R_{55}\sin k_3(x-M_3)+R_{56}\cos k_3(x-M_3), \quad M_3-L_{31}\leqslant x<M_3+L_{32} \\ R_{61}e^{BK_1(x-M_3-M_2/2)}+R_{62}e^{-BK_2(x-M_3-M_2/2)}+R_{63}e^{K_1(x-M_3-M_2/2)} \\ \quad +R_{64}e^{-K_2(x-M_3-M_2/2)}+R_{65}e^{-b_{32}(x-M_3-L_{32})}+R_{66}e^{b_{11}(x-M_2-M_3+L_{11})}, \\ \quad M_3+L_{32}\leqslant x<M_2+M_3-L_{11} \end{cases} \tag{62}$$

$$
U_0(x)=\begin{cases}
S_{11}e^{K_1(x+M_1)}+S_{12}e^{-K_2(x+M_1)}+S_{13}e^{BK_1(x+M_1)}+S_{14}e^{-BK_2(x+M_1)}+S_{15}\sin k_1(x+M_1) \\
\quad +S_{16}\cos k_1(x+M_1)+S_{17}e^{-\varepsilon(x+M_1)}, \quad -(M_1+L_{11})\leqslant x<-(M_1-L_{12}) \\
S_{21}e^{K_1(x+M_1/2)}+S_{22}e^{-K_2(x+M_1/2)}+S_{23}e^{BK_1(x+M_1/2)}+S_{24}e^{-BK_2(x+M_1/2)} \\
\quad +S_{25}e^{-b_{12}(x+M_1-L_{12})}+S_{26}e^{b_{21}(x+L_{21})}+S_{27}e^{-\varepsilon(x+M_1/2)}, \\
\quad -(M_1-L_{12})\leqslant x<-L_2 \\
S_{31}e^{K_1x}+S_{32}e^{-K_2x}+S_{33}e^{BK_1x}+S_{34}e^{-BK_2x}+S_{35}\sin k_2x+S_{36}\cos k_2x+S_{37}e^{-\varepsilon x}, \\
\quad -L_{21}\leqslant x<L_{22} \\
S_{41}e^{K_1(x-M_3/2)}+S_{42}e^{-K_2(x-M_3/2)}+S_{43}e^{BK_1(x-M_3/2)}+S_{44}e^{-BK_2(x-M_3/2)} \\
\quad +S_{45}e^{-b_{22}(x-L_{22})}+S_{46}e^{b_{31}(x-M_3+L_{31})}+S_{47}e^{-\varepsilon(x-M_3/2)}, \\
\quad L_{22}\leqslant x<M_3-L_{31} \\
S_{51}e^{-K_1(x-M_3)}+S_{52}e^{-K_2(x-M_3)}+S_{53}e^{BK_1(x-M_3)}+S_{54}e^{-BK_2(x-M_3)}+S_{55}\sin k_3(x-M_3) \\
\quad +S_{56}\cos k_3(x-M_3)+S_{57}e^{-\varepsilon(x-M_3)}, \quad M_3-L_{31}\leqslant x<M_3+L_{32} \\
S_{61}e^{K_1(x-M_3-M_2/2)}+S_{62}e^{-K_2(x-M_3-M_2/2)}+S_{63}e^{BK_1(x-M_3-M_2/2)}+S_{64}e^{-BK_2(x-K_3-M_2/2)} \\
\quad +S_{65}e^{-b_{32}(x-M_3-L_{32})}+S_{66}e^{b_{11}(x-M_2-M_3+L_{11})}+S_{67}e^{-\varepsilon(x-M_3-M_2/2)}. \\
\quad M_3+L_{32}\leqslant x<M_2+M_3-L_{11}.
\end{cases} \tag{63}
$$

Then the complete solutions of Eqs. (5)—(8) are obtained as $Q(x,y)=F_2(x)D_2(y)$.

As the heating function has the form

$$Q(x,y)=F_0(x)D_0(y)+F_1(x)D_1(y)+F_2(x)D_2(y), \tag{64}$$

we can simply add the solutions obtained in the above three cases to get all the solutions.

IV. SIMULATIONS OF CLIMATOLOGICAL FIELDS

In the present paper, climatological temperature are adopted from the atlas edited by Newell et al. (1972). Observed data of sea level pressure come from Crutcher's and Davis' atalas (1969).

In order to obtain the distributions of heat sources, the underlying climatological field from June to August (Fig. 1a) is referred. From Fig. 1a it can be seen that there are the following features of the climatological temperature field in summe (June—August).

(1) Along the South American coast of the Pacific Ocean, SST is rather low owing to the upwelling of sea water. There are also similar low temperature regions along the South African coast and near Australia. As it is winter in the Southern Hemisphere during June, July and August, the temperature is lower anywhere.

(2) In the Northern Hemisphere, the high temperature regions are in Sahara, Qinghai-Xizang Plateau and Mexico Gulf. The warm water in western Pacific moves to the Northern Hemisphere.

According to the features of temperature distribution in both hemispheres mentioned above, the heat sources (sinks) in summer are given in Fig. 1b. We can simulate the climatological distributions of p, u and v fields in the tropics and subtropics by the analytical solutions obtained in Section III.

The observed sea level air pressure in summer is shown in Fig. 2a and the simulated result is shown in Fig. 2b. The latter is very similar to the former.

The climatological surface temperature distribution in winter (December—February) is shown in Fig. 3a. Their characteristics are as follows:

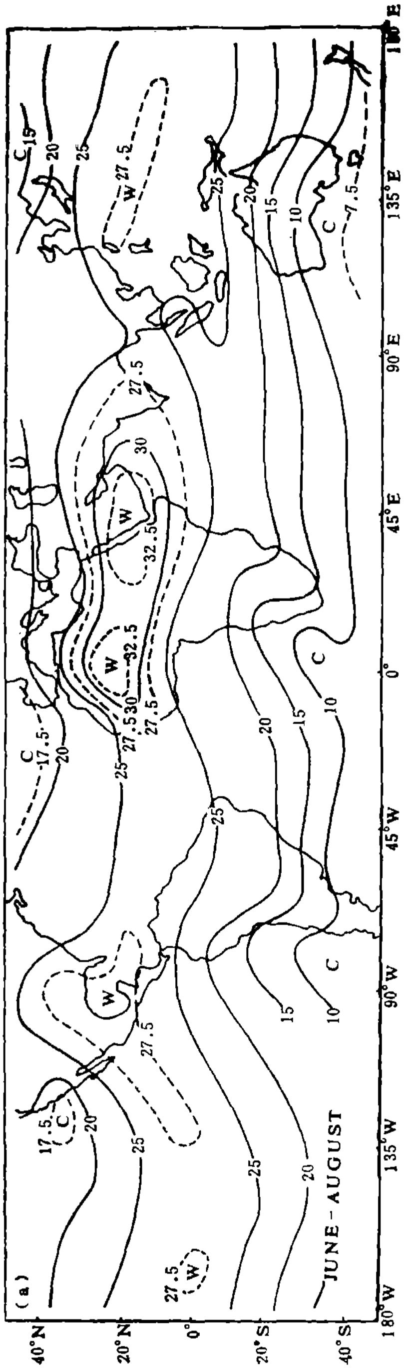

Fig. 1a. The climatological surface temperature distribution in the tropics and the subtropics in summer (June—August).

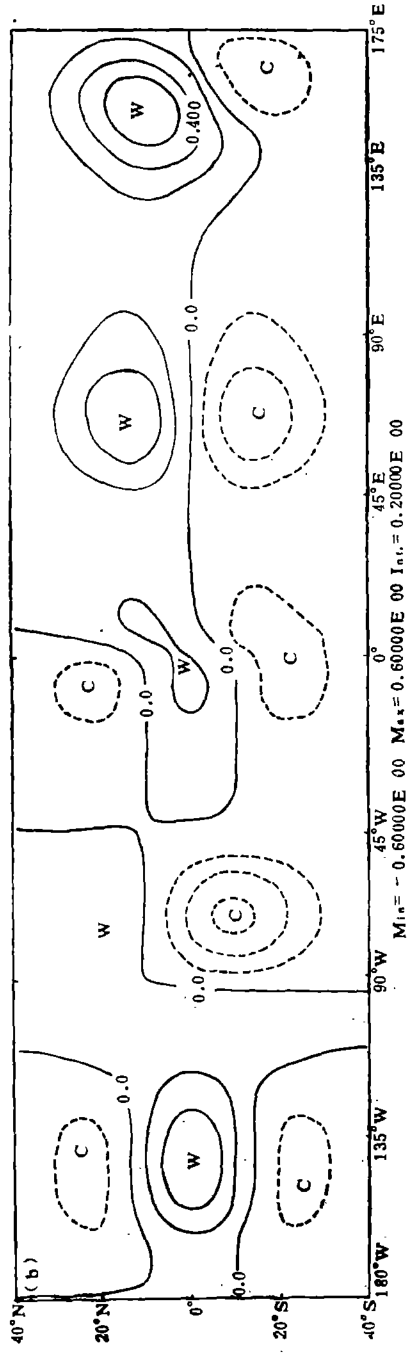

Fig. 1b. The distribution of heat source $Q(x,y)$ given in the tropics and the subtropics in summer.

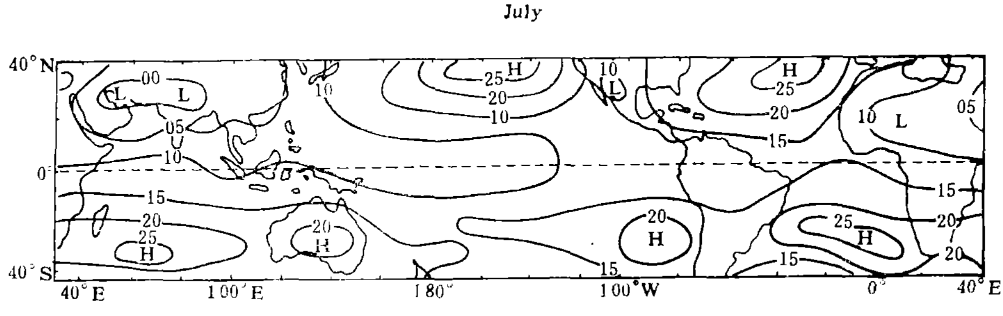

Fig. 2a. The climatological air pressure p at the earth's surface in July.

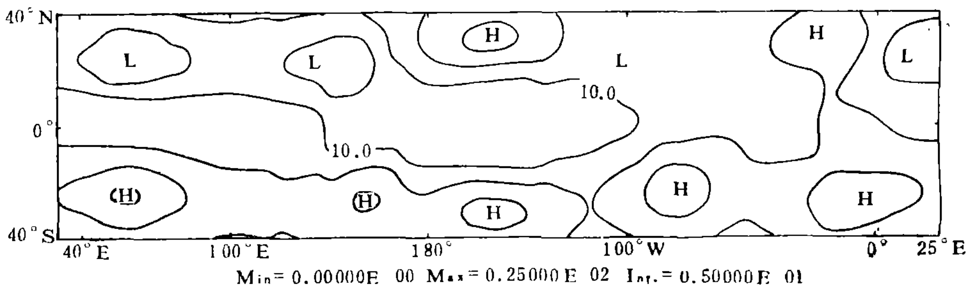

Fig. 2b. The simulated p in summer.

(1) There are still cold water regions along the South American coast of the Pacific Ocean and the South African coast of the Atlantic Ocean. As it is summer in the Southern Hemisphere during December, January and February, the warm water near Indonesia region moves to the Southern Hemisphere. A low temperature area appears in southern Indian Ocean.

(2) There are low temperature regions in the North America and in eastern Asia. As it is winter in the Northern Hemisphere at that time, the underlying temperature is relatively low. The warm water is mainly in the equatorial regions.

According to the features described above, the heat sources(sinks) considered in our model are shown in Fig. 3b. The observed air pressure at the earth's surface is shown in Fig. 4a and the simulated air pressure in Fig. 4b; It can be seen that the main features of observed data are described in simulated fields.

The climatological wind fields of the lower atmosphere are also simulated by using the present model (Ji, 1987). It is shown that the main features of observed fields are drawn in simulated fields. But as compared the simulation with the observation, there are still some discrepancies. Generally speaking, the westerlies are too strong in equatorial regions and somewhat weak in higher latitudes. If the zonal-averaged wind fields are not neglected in the model, we may get better results.

The climatological fields in spring and autumn are also simulated with this model (Ji, 1987). The results are not given here.

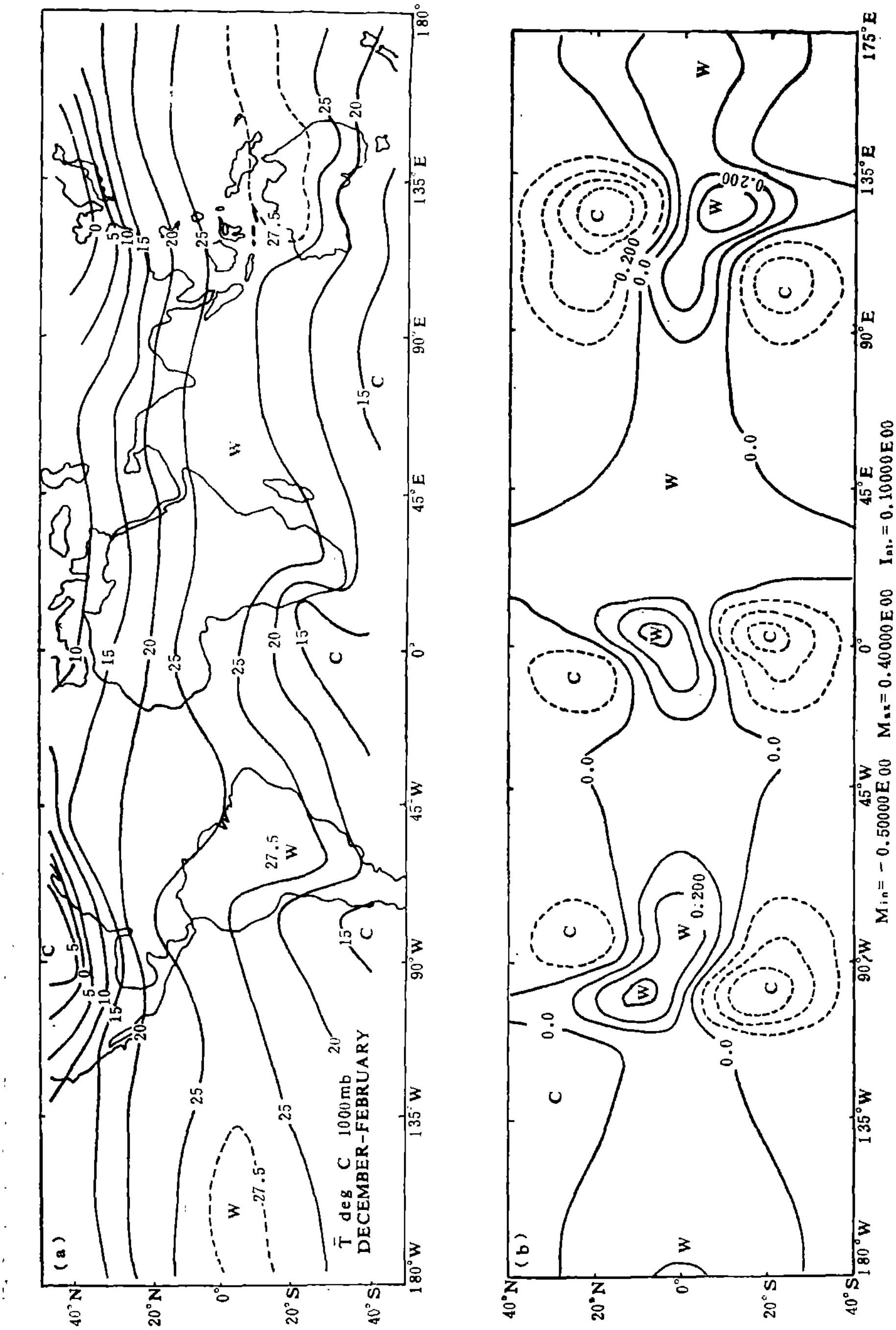

Fig. 3. As in Fig. 1a and 1b, except for winter (December—February).

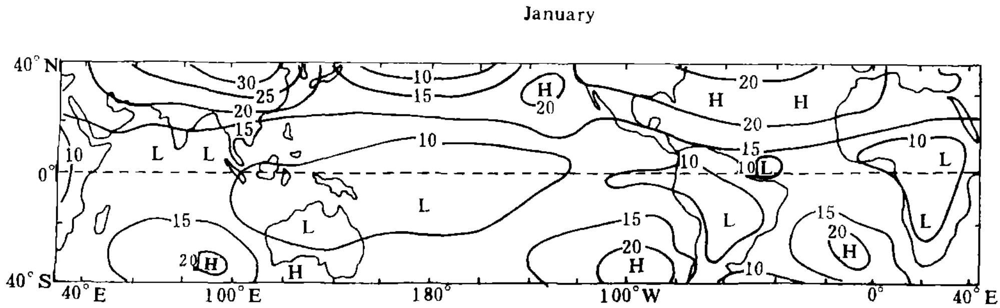

Fig. 4a. As in Fig. 2a, except for January.

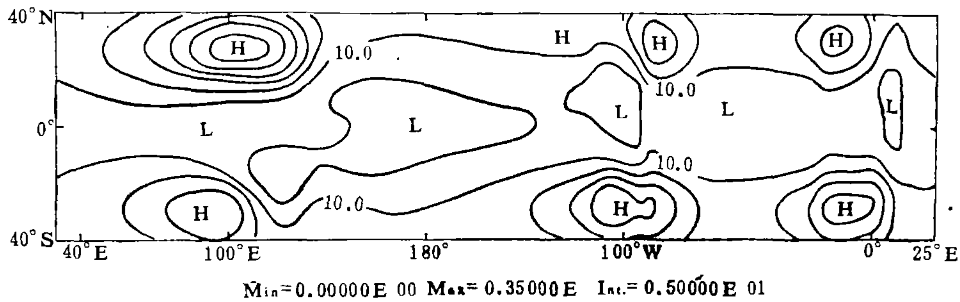

Fig. 4b. As in Fig. 2b, except for winter.

V. SUMMARY AND DISCUSSION

In the present paper, a linear steady model is constructed to investigate the response of the tropical atmosphere to diabatic heating. The basic equations are similar to those used by Gill (1980), but the long-wave approximation is removed and periodic boundary conditions are taken. Parabolic cylinder function $D_n(y)$ is utilized to solve the partial differential equations. The distribution function $F_n(x)$ in the x-direction of heat source $Q(x,y)$ is described in six sections respectively. Analytical solutions are obtained as $Q(x,y)$ taking the first three terms of $D_n(y)$, i. e., $Q(x,y)=F_0(x)D_0(y)+F_1(x)D_1(y)+F_2(x)\times D_2(y)$.

According to the features of the underlying surface temperature (including oceans and land), the heat sources(sinks) are given. Using this analytical model, we have simulated the climatological fields of winds and air pressure in the lower tropical and subtropical atmosphere in summer (June—August) and winter (December—February). The main features of observations are depicted in simulated fields. If the climatological fields can be simulated with such a simple model, it seems to be an encouragement to develop a simple numerical model for the prediction of tropical atmospheric circulation anomalies. This kind of model is being developed.

It is worthy to be mentioned that after the initial manuscript was finished, we noticed a recent work by Johnson et al. (1987). They gave global diabatic heating rates computed from ECMWF Level III analyses for 1979, which are coincident with Figs. 1b and 3b in this paper very well. Though the results of Johnson et al. are only for 1979, it is still shown qualitatively that our consideration for the heat sources (sinks) is reasonable in lower latitudes.

There are some systematic errors in the simulations, since the annual and zonal averaged background fields are omitted. In an effort to reduce at least some of the present model's systematic errors, the zonal mean wind should be taken into account. As we have discussed in Section II, the high (low) temperature area may not correspond to a heat source (sinks) in some cases. This problem should be investigated further. The distributions of ocean-land, orography and moisture air dynamics are not considered in the present model actually, and the nonlinear process is neglected in our model. However, all the above effects are important for understanding the activities of the tropical atmosphere. In fact, due to the limitation of mathematical methods, we can hardly take all these effects into account. It is necessary to clarify physical mechanisms of tropical circulations with analytical model. But if we attempt to consider all the processes in the atmosphere with one model, and its resolution is required very high, numerical general circulation models should be employed.

We would like to thank Prof. Chen Longxun for his useful suggestions and Prof. J. M. Wallace of University of Washington for his valuable comments, which helped to clarify and improve the manuscript.

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