# The Effects of Line-Wing Cutoff in LBL Integration on Radiation Calculations<sup>\*</sup>

ZHANG Hua<sup>1†</sup>(张 华), SHI Guangyu<sup>2</sup>(石广玉), and LIU Yi<sup>3</sup>(刘 毅)

1 Laboratory for Climate Studies of China Meteorological Administration, National Climate Center, Beijing 100081

2 State Key Laboratory of Numerical Modeling for Atmospheric Sciences and Geophysical Fluid Dynamics (LASG),

Institute of Atmospheric Physics, Beijing 100029

3 Laboratory for Middle Atmosphere and Global Environment Observation (LAGEO), Institute of Atmospheric

Physics, Beijing 100029

(Received March 10, 2008)

#### ABSTRACT

There are three basic methods in radiative transfer calculations, i.e., line-by-line (LBL) integration, correlated k-distribution method, and band model. The LBL integration is the most accurate of all, in which, there are two quadrature algorithms named in this paper as integration by lines and by sampling points when calculating atmospheric transmittance in the considered wavenumber region. Because the LBL integration is the most expensive of all, it is necessary and important to save calculation time but increase calculation speed when it is put into use in the daily operation in atmospheric remote sensing and atmospheric sounding. A simplified LBL method is given in this paper on the basis of integration by lines, which increases computational speed greatly with keeping the same accuracy. Then, we discuss the effects of different cutoff schemes on atmospheric absorption coefficient, transmittance, and cooling rate under both of accurate and simplified LBL methods in detail. There are four cutoff schemes described in this paper, i.e., CUTOFFs 1, 2, 3, and 4. It is shown by this numerical study that the way to cut off spectral line-wing has a great effect on the accuracy and speed of radiative calculations. The relative errors of the calculated absorption coefficients for CUTOFF 2 are the largest under different pressures, while for CUTOFF 1, they are less than 2% at most of sampling points and for CUTOFFs 3 or 4, they are almost less than 5% in the calculated spectral region, however, the calculation time is reduced greatly. We find in this study that the transmittance in the lower atmosphere is not sensitive to different LBL methods and different cutoff schemes. Whereas for the higher atmosphere, the differences of transmittance results between CUTOFF 2 and each of other three cutoff schemes are the biggest of all no matter for the accurate LBL or for the simplified LBL integrations. By comparison, the best and optimized cutoff scheme is given in this paper finally.

Key words: LBL (line-by-line) integration, line-wing cutoff, absorption coefficient, transmittance, cooling rate

#### 1. Introduction

LBL (line-by-line) integration is an exact method to calculate atmospheric transmittance with the contribution of atmospheric absorption with line by line. At present, with the rapid development of computer technology, it is not difficult to carry out LBL integration theoretically using the HITRAN database (Rothman et al., 2004), but the problem is calculation time. When LBL integration is put into use in the operation in atmospheric remote sensing and atmospheric sounding, it still is very important to save calculation cost and increase calculation speed. However, so far, LBL integration, the most exact method of radiative transfer, also cannot be directly used in climatic models even though the most advanced computer in the world is used (Zhang et al., 2000, 2005; Shen et al., 2004). Some similar methods based on LBL integration, such as band model and correlated k-distribution (CKD, for short) method, have to be used to enable LBL integration to be indirectly applied in climatic models (Zhang et al., 2005). Therefore, LBL model is the basis for the radiation scheme applied in climatic models and the standard reference for the final result

<sup>\*</sup>Supported by the National Natural Science Foundation of China under Grant No. 40775006, the National Basic Research Program of China under Grant No. 2006CB403707, and the Public Meteorology Special Foundation of China under Grant No. GYHY200706036.

<sup>&</sup>lt;sup>†</sup>Corresponding author: huazhang@cma.gov.cn.

comparison. In order to save calculation time, the most basic and traditional integration method is to use the grid points with a changeable wavenumber step. The quadrature scheme proposed by Kunde and Maguire (1974) is a typical of this method. However, CKD method requires an integration step with an equally spaced wavenumber, the above quadrature scheme cannot be used in the CKD method.

Usually, the factors that have influences on the accuracy and efficiency of the calculation of absorption coefficient in LBL integration include the following aspects: (1) the selection of integration step and sampling point; (2) the line-wing cutoff scheme, what are related to this are the contribution of spectral lines outside the considered wavenumber region and the neglect of weak lines within the region; and (3) there are three line types including Lorentz, Doppler, and Voigt that are suitable for different atmospheric pressures. Voigt function is commonly used as a popular method for the whole atmosphere (Clough et al., 1992; Clough and Iacono, 1995). It will take much more time to calculate Voigt function, which will change into the wing part of Lorentz Profile in the place far from the line center. Therefore, Voigt function can be used in a region near to the line center, whereas in other parts of the line, the simplified Lorentz function can be used for the calculation. Generally, a value of 200 fixed as the dividing point between Voigt and Lorentz functions could satisfy the need for the increase in calculation speed with a defined accuracy (Zhang and Shi, 2000). The calculation results show that the different line-wing cutoff is the most important one of the three factors above and the selection of an integrated step is important only to the lower atmosphere (Zhang, 1999).

Many researchers adopted a fixed wavenumber cutoff scheme (Clough et al., 1992; Clough and Iacono, 1995; Lacis and Oinas, 1991; Fu and Liou, 1992). Although it is simple to use, the negligible contribution from weak lines and the line wing part from strong lines in the higher atmosphere is calculated unnecessarily to make the cost increase, whereas in the lower atmosphere the contribution from strong line-wings may not be given enough consideration. Zhang and Shi (2000) put forward a new cutoff scheme on the basis of spectral line strength, however, they did not answer which scheme is the best one in the LBL integration.

In this paper, the integration method by lines is presented for an integrated step with equally spaced wavenumber, and a simplified LBL integration has been given on the basis of this. The effect of different line-wing cutoffs on the calculation of absorption coefficient, atmospheric transmittance and cooling rate with both of accurate and simplified LBL methods is emphasized in this work. And then the optimal cutoff scheme is given according to this study finally.

# 2. LBL methods

#### 2.1 Accurate LBL method

In recent years, many researchers (Lacis and Oinas, 1991; Fu and Liou, 1992, etc.) adopted a fixed value of 5 cm<sup>-1</sup> from line center to cutoff line-wing in LBL integration. To make a comparison with these works, a fixed 5-cm<sup>-1</sup> cutoff is adopted in this work as one of the cutoff schemes and the strict wavenumber integration is used according to the following Eq.(1) as a comparative standard reference for the calculations in this paper. Of these, Voigt function is applied into the whole atmosphere (Humlicek, 1982).

To average transmittance written as  $\overline{T}(u)$  over wavenumber interval  $\Delta v$ , the strict wavenumber integration according to the following equation is called accurate LBL integration,

$$\overline{T}(u) = \frac{1}{\Delta v} \int_{\Delta v} \exp(-k_v u) \mathrm{d}v, \qquad (1)$$

where the absorption coefficient of Lorentz line type  $k_v$  is

$$k_v = \frac{S}{\pi} \frac{\alpha_{\rm L}}{(v - v_0)^2 + \alpha_{\rm L}^2}.$$
 (2)

For Doppler line type,  $k_v$  is

$$k_v = \frac{S}{\alpha_{\rm D}\sqrt{\pi}} \exp[-(\frac{v - v_0}{\alpha_{\rm D}})^2].$$
 (3)

Whereas for mixed and widened Voigt line type,  $k_v$  is

$$k_v = Sf(v - v_0) = \frac{k_0 Y}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-t^2)}{Y^2 + (x - t)^2} dt, \quad (4)$$

where u is absorber amount;  $S, \alpha_{\rm L}, \alpha_{\rm D}, \text{and } v_0$ are the spectral line strength, half width of Lorentz and Doppler functions, and wavenumber of spectral line center, respectively, and  $k_0 = S/(\alpha_{\rm D}\sqrt{\pi})$ ,  $x = (v - v_0)/\alpha_{\rm D}, Y = \alpha_{\rm L}/\alpha_{\rm D}$ .

In the practical calculation of Eq.(1), the traditional method is used to carry out wavenumber (frequency) integration by sampling points, which is a method to use sampling points as the first integration loop and calculate the contribution of all spectral lines in the considered wavenumber region one by one. We use this method as the standard reference for the accurate LBL method in this paper. At the same time, a new integration method is presented in this paper, namely integration by lines, which is a method to use a spectral line as the first loop and calculate its contribution to all the sampling points in the considered wavenumber region simultaneously. Because the integration method by sampling points requires judging the range of the contributing spectral lines repeatedly, more calculation time is spent in the unparallel scalar calculation for this method, whereas the integration by lines can increase calculation speed significantly.

It should be noted here that the final accuracy of different LBL integrations could not be judged because of lack of complete and accurate laboratory measurements. Therefore, the accuracy comparison between LBL integrations can only be made among models. We make this "standard reference" only to compare it with the traditional LBL method, but this does not mean that it is the most accurate or has the highest efficiency.

## 2.2 Cutoff schemes

Theoretically, the limit of frequency integration is infinity in the calculation of absorption coefficient  $k_v$ . In other words, when  $k_v$  is calculated, the contribution of spectral lines at infinity should be considered. However, because there exist some errors and uncertainties to express the strength of spectral lines and the behavior of spectral lines far from the line center, it is actually insignificant to unlimitedly calculate the contribution from the wing of spectral lines far from the line center.

There are three approaches to cut the line-wing

Firstly, spectral lines are cut off within a fioff. xed wavenumber range, starting from the central wavenumber  $v_0$  of spectral lines. Lacis and Oinas (1991) and other researchers used this scheme, i.e., a fixed  $5 \text{ cm}^{-1}$  is taken as the cutoff wavenumber for each spectral line. This method is quite simple to use, but to strong absorption lines in the lower atmosphere, the contribution of line-wing absorption may be underestimated, whereas to the weak lines or in the higher atmosphere, this method may waste calculation time. Take the range of 665–675 cm<sup>-1</sup> of CO<sub>2</sub> 15  $\mu$ m for example, if the Doppler half-width  $\alpha_{\rm D} = 5 \times 10^{-4} \ {\rm cm}^{-1}$  in the higher atmosphere, it can be obtained by taking it into Eq.(3) that spectral line absorption is declining in accordance with  $\exp\{-[(v-v_0)/\alpha_D]^2\}$ . If  $v-v_0=5$  $cm^{-1}$ , the exponent value of the exponential function will be  $-10^8$ , meaning that absorption coefficient has become  $e^{-100000000}$  times of that at the line center, which is certainly unnecessary to consider. Secondly, the cutoff is made by a fixed multiple of a spectral line half-width. Shi (1981) and Chou and Arking (1980, 1981) used this scheme. The contribution of line-wing is cut off at a wavenumber away from the line center according to the smallest line strength of all the spectral lines and the requirement of accuracy within the considered spectral range. Supposing that  $v_c$  is cutoff wavenumber, and letting

$$v_c = \beta \alpha_{\rm L},\tag{5}$$

where  $\beta$  is an integer. To make up for the errors resulting from the cutoff itself partially, the corresponding line strength can be recorrected as (Zhang and Shi, 2000; Shi, 1998),

$$S' = S/[1 - 2/(\beta \pi)].$$
 (6)

The contribution of spectral line out of the range  $\Delta v$  can also be calculated in this way and this scheme is easy to use, but the contribution from weak linewings is also considered excessively so that the calculation time is wasted, however, the contribution from strong line-wings may be considered insufficiently by such a way.

Thirdly, Zhang and Shi (2000) proposed two new cutoff schemes for the line-wing contribution according to the different strengths of spectral lines in the considered wavenumber region. In the form of equation, both two schemes are similar to the second one described above, but  $\beta$  in Eq.(5) is not constant but changeable with the change of spectral line strength. In the first one, supposing that the smallest strength of spectral line in the range  $\Delta v$  is  $S_{\min}$ , if the strength of the *l*th spectral line is  $S_l$ , in order to keep the absorption coefficient with the same values not to be neglected for the *l*th spectral line, then we have

$$\beta = \beta_{\min} \sqrt{S_l / S_{\min}},\tag{7}$$

where  $\beta_{\min}$  is a prescribed constant, and then the line strength is also recorrected with Eq.(6). In the second one, supposing that the biggest strength of spectral line in the range  $\Delta v$  is  $S_{\max}$ , then, like getting Eq. (7), we obtain

$$\beta = \beta_{\max} / \sqrt{S_{\max} / S_l}, \tag{8}$$

where  $\beta_{\text{max}}$  is also a prescribed constant, and then the line strength is recorrected with Eq.(6) again.

## 2.3 Simplified LBL method

A simplified LBL method is obtained on the basis of the studies above. Its characteristics are as follows: it uses Eq.(1) for the wavenumber integration by lines; it cuts off the line-wings of all the spectral lines by different multiple of half-width according to spectral line strength; it uses the dividing point I=200 between Voigt and Lorentz functions to calculate absorption coefficient under all pressures and temperatures according to Zhang and Shi (2000); Voigt function is used to calculate absorption coefficient when i < I, while a simplified Lorentz function taking the form of Eq.(10)below is used to calculate absorption coefficient when i > I. Additionally, when the pressure p < 100 hPa, the integral step  $\delta_v = \alpha_{\rm D}$ , while it is taken as  $\delta_v = \alpha_{\rm L}$ when p > 100 hPa, and then the cubic spline function is used to make interpolation to get the absorption coefficient at the same number of sampling points.

Supposing that it starts from the line center  $v = v_0$ , the integral step on the axis of wavenumber (frequency) is  $\delta_v$ , the following method is used to select sampling points (Zhang, 1999; Shi, 1998),

$$M\delta_v = v - v_0 = M\alpha,\tag{9}$$



**Fig.1.** The schematic of region dividing for spectral lines.

where M is an integer and  $\alpha$  is expressed as Lorentz half-width  $\alpha_{\rm L}$  or Doppler half-width  $\alpha_{\rm D}$ , thus Eq.(2) is simplified to

$$k_v = \frac{S}{\pi \alpha_{\rm L}} (1 + U^2)^{-1}, \qquad (10)$$

where U = M, M < I = 200.

The schematic of spectral line region of simplified method is shown in Fig.1. In actual calculation,  $\Delta v$ can be divided into seven kinds of sub-ranges according to the location of spectral line (the right and left ends of the range are expressed as A and B; M is the total number of sampling points.  $M_L$  stands for the number of sampling point of each spectral line, and can be calculated from  $M_L = v_c / \beta_v$ . Here  $\delta_v = (B - A) / M$ is integral step and  $v_c = \beta_L \alpha$  is the cutoff wavenumber of spectral line L. As mentioned above, I is the dividing point to calculate absorption coefficient by using Voigt function and simplified Lorentz function (10); A' and B' stand for the two ends at 5 cm<sup>-1</sup> outside of the range [A, B]. Spectral line  $L_1, L_2, L_3, L_4, L_5, L_6$ , and  $L_7$  are located in the sub-ranges of [A', 1], [1, I], $[I, M_L], [M_L, M - M_L], [M - M_L, M], [M - I, M],$ and [M, B'], respectively. It is obvious that the seven different kinds of sub-ranges should be differentiated in the calculation process. In order to save calculation time of absorption coefficient by using Voigt function, we only calculate the right half part of absorption lines because the value in the left part can be obtained according to their symmetry.

#### 3. Results

# 3.1 The effect of different cutoff schemes on absorption coefficient

In this paper, we take the 500–800-cm<sup>-1</sup> region of CO<sub>2</sub> 15- $\mu$ m band for example. The simplified method is used to calculate the effects of the four cutoff schemes (expressed as CUTOFF) on absorption coefficient under different pressures. (1) CUTOFF 1:  $v_c=5$ cm<sup>-1</sup>, integration by lines is used in the considered regions; (2) CUTOFF 2: using Eq.(5), where  $\beta=300$ ; (3) CUTOFF 3: using Eq.(7), where  $\beta_{\min}=1$ ; and (4) CUTOFF 4: using Eq.(8), where  $\beta_{\max}=30000$ .

The results are shown in Fig.2. It indicates the percentage error of the absorption coefficient between those calculated by different cutoff schemes above with the simplified LBL method and by fixed  $5 \text{ cm}^{-1}$  cutoff with the accurate LBL method, in which solid and dashed lines are the results before and after the rearrangement of absorption coefficient, respectively. The results for the rearrangement of absorption coefficient are to be used in the correlated k-distribution method (Shi, 1981; Lacis and Oinas, 1991; Fu and Liou, 1992; Zhang et al., 2003), thus they are listed here for the reference. Taking p=10 hPa for example, for CUT-OFF 1, the largest error is 8% and mainly located at the right end, and the error is less than 2% at most of sampling points; for CUTOFF 2, the biggest error is 26%; for CUTOFFs 3 and 4, the largest error is 8%, and the error is less than 5% at most of sampling points. The results under other pressures are similar to those p=10 hPa (figures omitted). In the lower atmosphere, when p > 100 hPa, the percentage errors of the absorption coefficient is not sensitive to different cutoff schemes and the error distribution is totally the same (figures omitted). This is attributed to the adoption of interpolation of absorption coefficient when p > 100hPa. We can conclude by these comparisons that the errors of the calculated absorption coefficients are the largest for CUTOFF 2 under different pressures, indicating that the cutoff scheme used by previous researchers is not the optimized one. For CUTOFFs 3 or 4, all the percentge errors of absorption coefficients calculated at most sampling points are within 5%, especially those after the rearrangement of absorption coefficient. The errors become larger only for smaller absorption coefficient, but the time spent on the calculation of absorption coefficient with CUTOFFs 3 and 4 decreases significantly.

It is important to validate absorption coefficients in this study, but the most important is to validate the results of cooling rates with them, because a larger percentage error of smaller absorption coefficient does not necessarily have significant effect on the result of cooling rate. We will discuss the effects of different cutoff schemes on transmittance and cooling rate in the following section.

# 3.2 The effects of different cutoff schemes on transmittance and cooling rate

The following eight schemes to get absorption coefficient can be used to calculate cooling rate.

Method 1: in the accurate LBL integration, cutoff schemes are (1) CUTOFF 1, integrations over wavenumber are made by sampling points; (2) CUT-OFF 2; (3) CUTOFF 3; and (4) CUTOFF 4. Integrations over wavenumber are made by lines in (2), (3), and (4).

Method 2: in the simplified LBL integration, the four cutoff schemes are the same as those of Method 1, but when CUTOFF 1, integrations by lines over wavenumber should be used.

In Figs.3a, b, the effects of different cutoff schemes on transmittance are shown by using the accurate and simplified LBL methods above when p=10 hPa and T=260 K. Of these, the variability range of the absorber amount has been chosen to make the corresponding transmittance change from 0.0 to 1.0. It is shown in Fig.3 that the transmittances calculated by the two LBL methods and four cutoff schemes have been overlapped together in the lower atmosphere, indicating that the transmittance in the lower atmosphere is not sensitive to different LBL methods and cutoff schemes, which is consistent with the result of absorption coefficient mentioned above. In the higher atmosphere, no matter Methods 1 or 2 is used, the transmittance results from CUTOFF 2 are much different from those from other cutoff schemes, which is also consistent with the result of absorption coefficient.



**Fig.2.** The relative errors between absorption coefficients calculated by each of different cutoff schemes under simplified LBL methods and those by fixed cutoff of 5 cm<sup>-1</sup> under accurate LBL methods. The horizontal axis presents sampling points while the vertical axis is relative errors (unit: %).

It should be noted that the change of absorber amount in the real atmosphere is not as significant as that shown in Fig.3.

Taking CO<sub>2</sub> 665–675-cm<sup>-1</sup> region for example, the comparison of the cooling rate among the four cutoff schemes by using the two LBL methods is shown in Table 1. We can find in Table 1 that the accuracy of CUTOFF 4 is fairly high, all the absolute errors of cooling rate are less than 0.001 K day<sup>-1</sup> at all the levels of the atmosphere either for accurate or simplified LBL integration, while the calculation time is the least. The time can be saved by two orders of magnitude when CUTOFF 4 simplified LBL method is used; then the accuracy of CUTOFF 3 is also fairly high, but the time spent on the calculation is more than that of CUTOFF 4. It can be concluded from the above study that the optimized cutoff scheme is CUTOFF 4 for the atmospheric cooling rate.

### 4. Conclusions

In this paper, the effects of line-wing cutoff schemes on absorption coefficient, transmittance, and cooling rate are studied in detail by using both of accurate and simplified LBL methods. There are totally four cutoff schemes described in this work, i.e., CUTOFFs



Fig.3. The change of transmittance of different cutoff schemes with absorber under conditions of p=10 hPa and T=260 K and by use of (a) the accurate LBL method and (b) the simplified LBL method.

Scheme		1		2				
CUTOFF	1	2	3	4	1	2	3	4
CPU(s)	6808.0	2457.5	1144.8	949.2	1090.5	110.0	127.6	72.3
Height	C	$\Delta C_2$	$\Delta C_3$	$\Delta C_4$	$\Delta C_1$	$\Delta C_2$	$\Delta C_3$	$\Delta C_4$
(km)	$(K day^{-1})$							
60.83	-1.05958	1E-05	3E-05	0	0.00765	1E-05	5E-05	
58.62	-1.28569	0.00877	2E-05	5E-05	0.01263	3E-05	8E-05	
56.34	-1.54974	0.01376	3E-05	7E-05	0	0.02001	7E-05	0.00012
54	-1.87365	0.02076	1E-04	1E-04	-1E-05	0.03038	0.00018	0.00018
52.81	-2.06619	0.02522	0.0002	0.0001	-1E-05	0.03699	0.00028	0.00023
50.37	-2.45157	0.03565	0.0004	0.0002	-2E-05	0.05234	0.00068	0.00034
47.9	-2.26956	0.0451	0.0005	0.0002	-1E-05	0.06477	0.00098	0.00046
46.66	-2.02612	0.04893	0.0005	0.0003	-1E-05	0.06895	0.00096	0.00051
44.23	-1.54405	0.05622	0.0004	0.0003	-1E-05	0.07602	0.00083	0.00062
41.84	-1.13488	0.06338	0.0004	0.0004	-2E-05	0.08243	0.00063	0.00072
40.67	-0.97178	0.06703	0.0003	0.0004	-3E-05	0.08584	0.00051	0.00076
38.36	-0.71808	0.07451	0.0002	0.0005	-4E-05	0.09317	0.00031	0.00085
36.1	-0.54527	0.08131	0.0001	0.0006	-6E-05	0.09338	0.00014	0.00085
34.99	-0.4856	0.08432	0.0001	0.0006	-7E-05	0.08609	3E-05	0.00071
32.81	-0.392	0.07633	5E-05	0.0005	-8E-05	0.06802	-1E-04	0.00031
30.67	-0.30959	0.05446	-1E-05	0.0003	-9E-05	0.04797	-0.0001	5E-05
28.57	-0.23268	0.03601	-3E-05	6E-05	-9E-05	0.03139	-0.0002	-0.0001
26.51	-0.16454	0.0161	-3E-05	-4E-05	-8E-05	0.01298	-0.0001	-0.0002
22.49	-0.07753	-0.00986	-2E-05	-8E-05	-2E-05	-0.0105	-5E-05	-0.0001
20.51	-0.04317	-0.01328	-1E-05	-5E-05	2E-05	-0.0131	0	-4E-05
18.56	-0.01703	-0.00997	0	-3E-05	3E-05	-0.0095	2E-05	0
16.62	-0.00595	-0.00493	0	-1E-05	2E-05	-0.0045	2E-05	1E-05
13.65	0.00304	-0.00091	0	0	0	-0.0008	0	0
11.03	-0.00135	-2E-05	0	0	0	-1E-05	0	0
9.98	-0.00118	0	0	0	0	0	0	0
7.08	-0.00046	0	0	0	0	0	0	0
5.17	-0.00023	0.00001	0	0	0	0	0	0
3.1	-0.0001	0.00001	0	0	0	-1E-05	0	0
1.01	-0.00002	0	0	0	0	0	0	0
0.00	-0.00000	0	0	0	0	0	0	0

Table 1. The effect of different cutoff schemes on the accuracy and CPU time in cooling rate calculation

Here, C represents reference cooling rate with 5 cm<sup>-1</sup> cutoff and integration with sampling point loop.  $\Delta C_i$  (*i*=2, 3, 4 and *i*=1,2,3,4)(unit: K day<sup>-1</sup>) are differences of cooling rate between each of three cutoff schemes under accurate LBL method or each of four cutoff schemes under simplified LBL method and accurate LBL method with fixed cutoff of 5 cm<sup>-1</sup>, respectively.

1, 2, 3, and 4, respectively. It can be concluded by the comparison under different pressures that the error of absorption coefficient is the largest when CUTOFF 2 is used for calculation; when CUTOFF 1, all the percentage errors of absorption coefficient at most of sampling points are less than 2%; when CUTOFFs 3 or 4, the percentage error of absorption coefficient calculated at most sampling points is less than 5%, especially for the rearrangement case of absorption coefficient. Only for the smaller absorption coefficients, the errors become larger, but the calculation time spent on the absorption coefficient with the two LBL methods decreases significantly for CUTOFFs 3 and 4. The transmittance in the lower atmosphere is not sensitive to the different LBL methods and the different cutoff schemes. Whereas in the higher atmosphere, the differences of transmittance results between CUT-OFF 2 and each of other three cutoff schemes are significant no matter for the accurate LBL or for the simplified LBL integrations. It indicates that the cutoff scheme adopted by previous researchers is not the best one.

Finally, it can be concluded by the comprehensive analysis on the transmittance and cooling rate results that the optimized cutoff scheme is the one based on the biggest spectral line strength proposed in this work, i.e., CUTOFF 4.

#### REFERENCES

- Chou, M.-D., and A. Arking 1980: Computation of infrared cooling rates in the water vapor bands. J. Atmos. Sci., 37, 855–867.
- Chou, M.-D., and A. Arking, 1981: An efficient method for computing the absorption of solar radiation by water vapor. J. Atmos. Sci., 38, 798–807.
- Clough, S. A., and M. J. Iacono, 1995: Line-by-line calculation of atmospheric fluxes and cooling rates 2. Application to carbon dioxide, ozone, methane, nitrous oxide and the halocarbons. J. Geophys. Res., 100, 16519–16535.
- Clough, S. A., M. J. Iacono, and J.-L. Moncet, 1992: Line-by-line calculation of atmospheric fluxes and cooling rates: Application to water vapor. J. Geophys. Res., 97, 15761–15785.
- Fu Q., and K.-N. Liou, 1992: On the correlated k distri-

bution method for radiative transfer in nonhomogeneous atmospheres. J. Atmos. Sci., 49, 2139–2156.

- Humlicek, J., 1982: Optimized computation of the Voigt and C complex probability functions. J. Quant. Spectrosc. Radiat. Transfer, 27, 437–444.
- Kunde, V. G., and W. C. Maguire, 1974: Direct integration transmittance model. J. Quant. Spectrosc. Radiat. Transfer, 14, 803–817.
- Lacis, A. A., and V. A. Oinas, 1991: Description of the correlated k distribution method for modeling nongray gaseous absorption, thermal emission and multiple scattering in vertically inhomogeneous atmospheres. J. Geophys. Res., 96, 9027–9063.
- Rothman, L. S., C. P. Rinsland, A. Goldman, et al., 2004: The HITRAN molecular spectroscopic database and HAWKS (HITRAN Atmospheric Workstation). http://www.hitran.com/, Updated HITRAN'2004.
- Shen Yuanfang, Huang Liping, Xu Guoqiang, et al., 2004: The sensitivity of long wave radiation of atmospheric changes and the simulating in the weather research and forecast (WRF) model. Acta Meteorologica Sinica, 62(2), 213–227. (in Chinese)
- Shi Guangyu, 1981: An accurate calculation and representation of the infrared transmission function of the atmospheric constituents. Ph. D. thesis, Tohoku University of Japan.
- Shi Guangyu, 1998: On correlated k-distribution model in radiative calculation. J. Atmos. Sci., 22, 659– 676. (in Chinese)
- Zhang H., T. Nakajima, Shi G.-Y., et al., 2003: An optimal approach to overlapping bands with correlated k-distribution method and its application to radiative calculations. J. Geophys. Res., 108, D20, 4641, doi: 10.1029/2002JD003358.
- Zhang Hua, 1999: On the study of a new correlated kdistribution method for nongray gaseous absorption in the inhomogeneous scattering atmosphere. Ph.D. thesis, the Institute of Atmospheric Physics, Beijing, China. (in Chinese)
- Zhang Hua and Shi Guangyu, 2000: A fast and efficient algorithm of line-by-line integration for atmospheric absorption. J. Atmos. Sci., 24(1), 111–121. (in Chinese)
- Zhang Hua, Shi Guangyu, and Liu Yi, 2005: A comparison study between the two line-by-line integration algorithms. J. Atmos. Sci., 29(4), 581–594. (in Chinese)