THE GENERALIZED LIAPUNOV STABILITY FOR ATMOSPHERIC SYSTEMS

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ABSTRACT

The stability theory that describes the local stability of atmospheric systems is set up by the generalized Liapunovian second method on the basis of the nonequilibrium statistical physics. A combined hydro-thermodynamic stability criterion for the atmosphere is derived in light of the constructed generalized Liapunov functional which is suitable to describing the atmospheric system defined by the system of partial differential equations, and the concept and criterion of the hydro-thermodynamic stability are first introduced into the atmospheric thermodynamics, thus many ways of atmospheric motions with the background of macroscopic thermodynamics are explained.

I. INTRODUCTION

The theory on motion stability in Liapunovian sense deals with the influence of the disturbance factors on the motions of material systems (Qin et al., 1981a; Xu, 1962). A system that loses its stability near critical values is very sensitive to perturbations so that it would evolve to a certain eventual state, maybe considerably different from the initial state undisturbed. The atmosphere is a many-body system whose disturbant factors always exist inevitably. Therefore, the problem of the stability of atmospheric motions has been paid great attention to (Kuo, 1949; Miles, 1961; Haward, 1961; Fjörtoft, 1950; Green, 1960; Wu 1964a; Burger, 1962; Zeng, 1979; Yang et al., 1983; and Pedlosky, 1979). However, some of previous works completed by the linearized method to seek for characteristic values usually deal with the specific objectives and, the methods employed most belong to the category of Liapunovian first method. Moreover, there is something obviously unreasonable in the results obtained from the linearized method of normal modes, because there does not exist such a process which is globally stable or unstable, for instance, in the atmosphere and it is hard to image that the global (e.g. the entire earth or Northern Hemisphere) atmosphere is wholly stable or augments with time infinitely and monotonously. On the other hand, although it is noticed that the baroclinic instability is one kind of the forms of thermodynamic convection (Pedlosky, 1979), these works are not involved with the thermodynamic stability of the atmosphere in general. In fact, the atmospheric heat-engine is a complicated thermodynamic system and, it is often in the substable or weak-stable states (Prigogine, 1980). Meanwhile, this system also possesses the so-called intrinsic stability determined by the dissipativity according to the second principle of classical thermodynamics (Glansdorff and Prigogine, 1971). These problems, even for the thermodynamics and statistical physics itself, are still not matured. Now, on account of the progress of nonlinear nonequilibrium statistical physics and the preliminary success in the generalization of Liapunov stability theory setting up for ordinary differential equations to partial ones, one could utilize and expand the concept of Liapunovian second method, and construct, directly through a generalized Liapunov functional, a combined thermodynamic and dynamic stability (static stability included) criterion, by which one could systematically discuss the hydro-thermodynamic stability of the nonlinear atmospheric systems.

II. THE CONCEPT OF STABILITY

1. *The Definition of Stability*

The concept on Liapunov stability of motions is the direct generalization of that for equilibrium states of, e.g., a single pendulum in the classical mechanics (see, Qin et al., 1981a). However, the atmospheric system being often in weak states, would not possess the simplicity that is displayed in the problems about the stability of motion of rigid body with less freedom degrees. Therefore we will introduce another definition of stability when discussing the atmospheric many-body system.

The above-mentioned equilibrium or so-called reference state must be first undergone in defining stability. Any present motion state can be regarded as a certain kind of departure from a reference state. If the present motion tends to regress to the corresponding reference state, such a motion would be considered as stable; if it continues to depart from the reference state, considered as unstable. In the physics, consequently, this kind of reference state should be "the oscillatory center" of instantaneous states, or the so-called most probable state in terms of the statistical physics.

According to the Einsteinian fluctuation formula, the probability by which a certain fluctuation (disturbance) occurs can be estimated as

$$P_r \sim \exp\left[\frac{1}{2}\frac{\delta^2 s}{k}\right],$$

where $\delta^2 s$ is the second-order variation of entropy, k Boltzmann constant; as will be proved below, $\delta^2 s$ is the negative definite function of the fluctuation δA_k of independent variables A_k (e.g. the fluctuation of temperature δT, where "δ" denotes the departure from reference states); it is easily seen that the state with minimum fluctuations ($\delta A = 0$) has the maximal probability. Similarly, for a non-static fluid system ($v \neq 0$), its reference should be the state corresponding to $\delta v = 0$. Obviously, such a state is just a set of particular solutions (trivial ones) to the system of equations governing disturbances (δA_i).

Incidentally, it is because $\delta^2 s$ has so clear physical meaning that we choose $L \equiv \delta^2(\rho s) - T^{-1}\rho(\delta v)^2$, where ρ is the density and $\delta^2(\rho s) = \rho\delta^2 s$ (Glansdorff and Prigogine, 1971), as the preferable generalized Liapunov functional from many functions in discussing the stability by use of the direct method.

To sum up, as discussed above, the references are the most probable states with $\delta A_i = 0$, i.e., the "equilibrium states" around which the values of physical quantities at spatial points fluctuate. Therefore, in general cases, the climatic averaged states are approximately taken to be references.

2. *System of Governing Equations for Disturbances*

The starting equations are the system of equations which dominate the change of disturbances since what we are concerned with is whether or not the disturbances would grow. These equations can be derived from the general conservative laws of mass, momentum and energy through variation operations.

They are the motion equations for disturbances

$$\frac{\partial}{\partial t}[\delta(\rho v_i)] = \delta(\rho F_i) - \frac{\partial(\delta p)}{\partial x_i} - \mathrm{div}\,\delta(\rho v_i \boldsymbol{V}), \qquad (i=1,2,3) \tag{1}$$

the continuity equation

$$\frac{\partial}{\partial t}[\delta\rho] = -\mathrm{div}\ \delta(\rho \boldsymbol{V}), \tag{2}$$

the mass-conservative equation

$$\frac{\partial}{\partial t}[\delta\rho_r] = \sum_k \nu_{rk} M_r \delta\omega_k - \mathrm{div}\ \delta(\rho_r \boldsymbol{V}), \tag{3}$$

and the first law of thermodynamics

$$\frac{\partial}{\partial t}[\delta(\rho e)] = -\mathrm{div}\ \delta(\rho e \boldsymbol{V}) - \delta(p\ \mathrm{div}\ \boldsymbol{V}) - \mathrm{div}(\delta \boldsymbol{W}), \tag{4}$$

where v_i denotes the component velocity in the x_i direction in Cartesian coordinates; F_i is the external force exerted in the x_i-direction, including the noninertial force, e.g. the Coriolis force; ρ_r is the partial density of component $r\,(\sum_r \rho_r = \rho)$; $\sum_k \nu_{rk}\ M_r\ \delta\omega_k$ is representative of the contribution of all the k chemical reactions to the density change of component r, in which the influence of moisture phase transition (δc, condensation rate) can be retained for our problems treated here; e is the specific internal energy that can be approximately expressed as $C_v T$ in the range of atmospheric temperature with C_v being the specific heat at constant volume; W is the heat flow vector; and others are the commonly used symbols.

3. *Liapunovian Direct Method and Its Reestablishment*

First, we briefly review the concept of Liapunovian direct method, and then pose our method to discuss the stability.

The key point of the Liapunovian second method is to introduce the definite Liapunov functional L. The theorem of stability proved by Liapunov shows that the stability of a system depends on the sign of $L\partial L/\partial t$ (Qin et al., 1981a).

However, as mentioned above, the definition of Liapunov stability originates from the classical mechanics. On the other hand, because of the complicated feedback effects of various factors in the atmosphere, it is impossible that the stability of instantaneous disturbances in the atmosphere is so simple as shown by the linear normal mode method in which it infinitely increases from or monotonously regresses to, the fundamental state with time; and that all the disturbances simultaneously undergo global growth or decay. In fact, they alternatively grow and decay. Therefore, it is meaningful in practice to solve the problem about what direction the present instantaneous states developed from (small) disturbances will evolve in, either continuing to depart from or regressing to the fundamental states (reference ones). Obviously, it turns out the definition of stability in Paragraph 2.

Now we will transform the Liapunovian second method into the direct method suitable to our concept of stability.

First, Liapunov functional should be the function of disturbance variables $\delta A_k (k=1,2, \cdots, n$; n is the number of independent variables) since the problem under consideration is whether the disturbance grows or not. Second, L should be a definite function (assuming to be negative). The simplest form for the function which meets the above two requirments can be taken as

$$L=-\sum_{k=1}^{n}\alpha_k(\delta A_k)^2, \tag{5}$$

where α_k are positive coefficients. It is easily seen that L in Eq. (5) is similar in form to the distance in Hilbert space. The only difference between them is that L is a "negative distance" with a weight. Thus, according to our definition of stability, the present states must regress to the reference state for a stable system, and the distance will inevitably shorten with time, i.e., the "negative distance" will enlarge with time. As a result, we have

$$\frac{\partial L}{\partial t}=-\sum_{k=1}^{n}\alpha_k\frac{\partial}{\partial t}(\delta A_k)^2>0, \tag{6}$$

or

$$L\frac{\partial L}{\partial t}<0. \tag{7}$$

This is completely the same as the Liapunov stability criterion in form, but in content it is not the stability in Liapunovian sense. The function of this criterion is to judge whether or not local motions regress to references, and this stability is called the generalized Liapunov stability.

III. GENERALIZED LIAPUNOV FUNCTION

According to the direct method generalized in the previous section, one needs to choose a generalized Liapunov function as a starting point for discussing stability. The generalized Liapunov function mentioned above is

$$L=\delta^2(\rho s)-T^{-1}\rho(\delta v)^2, \tag{8}$$

where $\delta^2(\rho s)$ is the second-order variation of the entropy per unit volume (s being the specific entropy), $T^{-1}\rho(\delta v)^2$ is the generalized kinetic energy of disturbance, and $(\delta v)^2=\sum_{i=1}^{3}(\delta v_i)^2$.

Let us prove that L is a definite function.

According to the Prigogine theory of nonequilibrium thermodynamics (Glansdorff and Prigogine, 1971), we have

$$\delta^2(\rho s)=\delta T^{-1}\delta(\rho e)-\sum_r\delta(\mu_r T^{-1})\delta\rho_r. \tag{9}$$

In addition, we have

$$\delta^2(\rho s)=\rho\delta^2 s, \tag{10}$$

where $\delta^2 s$ can be written as (Reichl, 1980)

$$\delta^2 s=-\frac{1}{T}\left[\frac{C_V}{T}(\delta T)^2+\frac{\rho}{\chi}(\delta\alpha)^2_{N_r}+\sum_{rr'}\mu_{rr'}\delta N_r\delta N_{r'}\right], \tag{11}$$

where e is the specific internal energy, α the specific volume, μ_r and N_r are the chemical potential and fractional mass of component r, respectively; C_V the specific heat at constant volume,

χ the coefficient of heat expansion and

$$\mu_{rr'}=\left(\frac{\partial\mu_r}{\partial N_{r'}}\right)_{T,\ p,\ (N_r)}. \tag{12}$$

Here subscripts denote the variables that keep unchanged in taking partial derivatives.

If the local equilibrium assumption which is reasonable to the mixed ideal gases like the atmosphere, and the Gibbs-Duhem stability conditions which are completely satisfied by the atmosphere (Reichl, 1980)

$$C_V>0, \qquad \chi>0 \quad \text{and} \quad \sum_{rr'}\mu_{rr'}\delta N_r\delta N_{r'}>0 \tag{13}$$

are taken, it is obvious that $\delta^2(\rho s)$ is a negative definite function. The first two conditions in (13) are representative of the thermal and mechanical stability, respectively, and the third condition $\sum_{rr'}\mu_{rr'}\delta N_r\delta N_{r'}$ describes such an irreversible spontaneous phenomenon of the intrinsic stability by virtue of which the mixed gases of many components with inhomogeneous density will inevitably cause diffusion and thus tend to homogenization.

On the other hand, we have (Glasdorff and Prigogine, 1971)

$$\rho(\delta v)^2=\delta^2\left(\frac{1}{2}\rho v^2\right),$$

and

$$\delta^2\left(\frac{1}{2}\rho v_i^2\right)=\delta v_i\delta(\rho v_i)-\delta\left(\frac{1}{2}v_i^2\right)\delta\rho\ . \tag{14}$$

Substituting (9) and (14) into (8) gives

$$L=\delta T^{-1}\delta(\rho e)-\sum_r\delta\left(\mu_r T^{-1}-\frac{1}{2}T^{-1}v^2\right)\delta\rho_r-T^{-1}\delta v_i\delta(\rho v_i), \tag{15}$$

where subscript i repeated in the same term means summing over all the three components ($i=1,2,3$) in Cartesian coordinates. L in (15) is just the generalized Liapunov function we have chosen.

IV. HYDRO-THERMODYNAMIC STABILITY CONDITIONS FOR ATMOSPHERIC SYSTEMS

The stability conditions are determined by the sign of $\partial L/\partial t$ since L is the definite function. Considering the independent variables of disturbances (δA_k) chosen above, the partial derivative of (15) with respect to time will be

$$\begin{aligned}\frac{\partial L}{\partial t}&=\sum_k\frac{\partial L}{\partial(\delta A_k)}\frac{\partial(\delta A_k)}{\partial t}\\&=\delta T^{-1}\frac{\partial}{\partial t}\delta(\rho e)-\sum_r\delta(\mu_r T^{-1})\frac{\partial}{\partial t}\delta\rho_r+T^{-1}v\delta v\frac{\partial}{\partial t}\delta\rho-T^{-1}\delta v_i\frac{\partial}{\partial t}\delta(\rho v_i).\end{aligned} \tag{16}$$

Substituting the governing equations (1)—(4) for disturbances into (16), we gain an explicit formulation for the stability criterion. After a series of primary operations. the general hydro-thermodynamic stability conditions are eventually obtained which include both thermal and dynamic descriptions as follows

$$J\equiv A-T^{-1}B\equiv\sum_i A_i-T^{-1}\sum_i B_i$$

$$
\begin{aligned}
&=\frac{C_v}{RT^2}\delta p\delta \boldsymbol{V}\cdot\nabla T-\delta\boldsymbol{V}\cdot\sum_r(\nabla s_r)\delta\rho_r+\frac{\delta p}{T}\mathrm{div}\ \delta\boldsymbol{V}\\
&+\frac{C_v}{RT^2}\delta T\boldsymbol{V}_h\cdot\nabla\delta p-\sum_r\delta s_r\boldsymbol{V}_h\cdot\nabla_h\delta\rho_r-\delta c\delta(s_v-s_w)\\
&-\frac{\delta\rho}{T}\delta u[v(2\Omega\sin\varphi)]+\frac{\delta\rho}{T}\delta v[u(2\Omega\sin\varphi)]\\
&+\frac{\delta\rho}{T}\delta w[g-u(2\Omega\cos\varphi)]+\frac{1}{T}\frac{\partial u}{\partial x}\delta u\delta(\rho u)+\frac{1}{T}\frac{\partial v}{\partial y}\delta v\delta(\rho v)\\
&+\frac{1}{T}\frac{\partial v}{\partial x}\delta v\delta(\rho u)+\frac{1}{T}\frac{\partial u}{\partial y}\delta u\delta(\rho v)+\frac{1}{T}\frac{\partial u}{\partial z}\delta u\delta(\rho w)\\
&+\frac{1}{T}\frac{\partial v}{\partial z}\delta v\delta(\rho w)+\frac{1}{T}(\delta v)^2\mathrm{div}\ \rho\boldsymbol{V}+\frac{1}{T}\delta\boldsymbol{V}\cdot\nabla\delta p\\
&+\frac{1}{T}\rho\boldsymbol{V}_h\cdot\nabla\frac{1}{2}(\delta v)^2, \qquad (<0,\ \text{unstable}) \qquad (17)
\end{aligned}
$$

where δv is the whole disturbance velocity, $\sum_r$ denotes summing over r components of system. Considered in the atmospheric systems are three components $r=d, v$ and w, respectively representative of the dry air, vapour and water-drop (liquid water). Symbol ",j" denotes $\partial/\partial x_j$. A stands for the part of thermal contributions and B the dynamic ones. In the derivation of (17), the irreversible dissipative effects caused by the diffusion in various components of atmospheric systems, chemical reactions, viscosity, heat conduction, etc. have been omitted. In addition, also used are the relation between chemical potential and enthalpy (Ma et al., 1982), thermodynamic relation for phase transition (Iribarne and Godson, 1973), state equation for ideal gases, entropy expression for mixed ideal gases, etc. For the atmospheric motions of larger scales, we have

$$w_{,j}\ \delta w\simeq 0 \text{ and } \mathrm{div}\ \boldsymbol{V}\simeq 0, \qquad (18)$$

and always

$$
\begin{aligned}
\delta\boldsymbol{F}\cdot\delta\boldsymbol{V}&=[\delta v(2\Omega\sin\varphi)-\delta w(2\Omega\cos\varphi)]\delta u\\
&-\delta v[\delta u(2\Omega\sin\varphi)]-\delta w[\delta g-\delta u(2\Omega\cos\varphi)]=0. \qquad (19)
\end{aligned}
$$

By use of the above expressions, the criterion (17) has, in fact, been a simplified formulation for the original one.

Expression (17) is just the basic criterion for testing the hydro-thermodynamic stability of the atmosphere: $J<0$ means that the system is unstable, that is, the present state will continue to depart from the reference state and the system will become more and more asymmetric, hence structures are formed. On the other hand, $J\geqslant 0$ means that the system is stable, that is, the present state will regress to the reference state and the system will become more symmetric, hence structures tend to disappear ($J>0$) or quasi-steady ($J=0$).

V. THE PHYSICAL MEANING OF STABILITY CRITERION AND ITS SYNOPTIC ILLUSTRATIONS

In this section we examine the physical conditions of instability caused by several terms in the criterion (17) and try to explain some well-known synoptic observations. It should be noted that the following qualitative discussion puts emphasis on interpreting the physical implication of every term. In case the stability sign of a specific system is to be determined, the total contribution from all the terms must be calculated. Indeed, it is convenient in practical use that of all the terms in (17) the major ones are determined in advance through scale analysis with respect to a certain specific system or situation.

1. *Term* A_1

$$A_1 \equiv \frac{C_v}{RT^2}\delta p \delta \boldsymbol{V} \cdot \nabla T \ .$$

Obviously, $A_1 < 0$ (unstable) demands that the sign of δp is different from $\delta \boldsymbol{V} \cdot \nabla T$, implying that a warm advection $(-\delta \boldsymbol{V} \cdot \nabla T > 0)$ is accompanied with a pressure increase ($\delta p > 0$, symbol "δ" denotes the departure of local physical variables from references),and vice versa. In this case, the departures of the corresponding physical variables from references tend to enlarge.

We now examine the characteristics of stability for such systems as blocking highs, typhoons, etc.

The reference wind field of blocking highs can be approximately taken as the belt-flow. Furthermore, since the component δv dominates, A_1 has an approximate form

$$A_1 \simeq \frac{C_v}{RT^2}\delta p \delta v \ \frac{\partial T}{\partial y} \ .$$

It can be seen from Fig.1 that the west part of the blocking high is stable ($A_1 < 0$) while the east part unstable ($A_1 > 0$). According to our definition of stability, in the west part of the blocking high and its neighbourhood (i.e. in all the regions with $\delta p > 0$) the high will strengthen while in the east part $|\delta p|$ tends to decrease. In our opinion, the reason why the blocking high—the carrier on the westerlies, which should actually travel eastwards—often presents a quasi-static state or even moves back westwards, is closely related to the characteristics of the stability distribution inside the high. Meanwhile, the stability analysis also shows that the developing blocking high has experinced itself the supersession of "west-growing and east-decaying" from its beginning, and that the phenomenological movement is not necessarily the same as the behaviour of system as a substance.

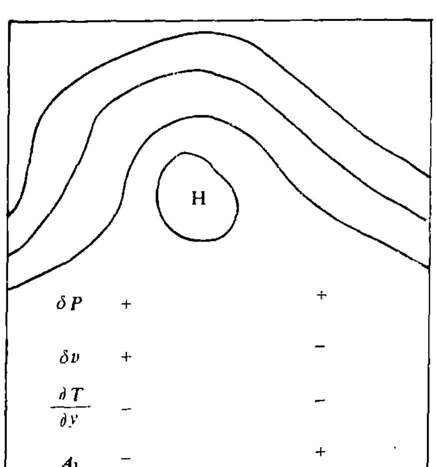

Fig. 1. The stability structure of blocking highs.

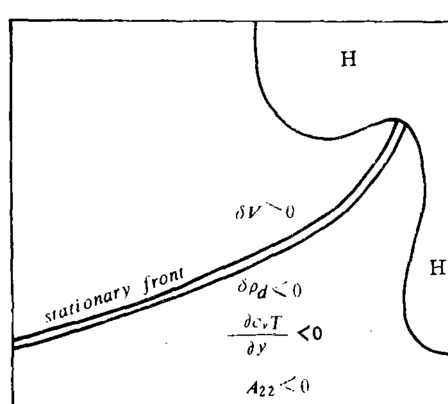

Fig. 2. The stability characteristics of the V-shaped inverted trough.

In brief, each term in the criterion (17) can be discussed in the way of term A_1. Synoptic meteorology provides us with abundant illustrations of stability. It is without difficulty to abstract the stability models of temperature, pressure, humidity and wind from the other terms of the criterion. Therefore, in order to avoid lengthy description, we only choose one or two cases to illustrate from conceptual sense.

2. *Term* A_2

$$A_2 \equiv -\delta V \cdot \sum_r (\nabla s_r)\delta\rho_r$$
$$= -\delta V \sum_r \left[\frac{C_{p_r}}{T}\nabla T - \frac{R}{p_r}\nabla p_r\right]\delta\rho_r.$$

A_2 can be decomposed into two terms, A_{21}, the vertical component, and A_{22}, the horizontal one.

It is easily demonstrated that A_{21} can be reduced to the common criterion of static stability when used in the single component (dry air). This is because for the dry air alone we have

$$\delta s \equiv \delta s_d = C_{pd}\frac{\delta T}{T} - R\frac{\delta p_d}{p_d}, \tag{20}$$

and then

$$A_{21} = -\delta w\left(\frac{C_p}{T}\frac{\partial T}{\partial z} - \frac{R}{p}\frac{\partial p}{\partial z}\right)\delta\rho$$
$$= \frac{C_p}{T}\left(r - \frac{g}{C_p}\right)\delta w\frac{\partial\rho}{\partial z}\delta z, \tag{21}$$

where subscript d has been omitted. Seeing that the density of dry air in the atmosphere generally decreases with height, i.e.

$$\frac{\partial\rho}{\partial z} < 0,$$

we immediately obtain (noting that $\delta w\delta z$ is identically positive)

$$r > r_d \longleftrightarrow \text{unstable.} \qquad (A_{21} < 0)$$

Also it is seen that A_{21} comprises more information than the static stability and shows that the unstable degree is proportional to the vertical velocity and vertical lapse rate of density.

In addition, for the single component d, A_{22} can be reduced to

$$A_{22} \approx \left[\left(-\frac{\delta V_h}{T}\cdot\nabla_h C_{pd}T\right) - \left(-\frac{\delta V_h}{T}\cdot\frac{1}{\rho_d}\nabla_h p_d\right)\right]\delta\rho_d. \tag{22}$$

It is seen that the first term in the brackets is representative of the sensible heat advection and, the second term the working rate of the pressure gradient force.

A_{22} shows that in the region where the density is smaller than the reference ($\delta\rho_d < 0$), the loss of stability would occur as long as the sensible heat advection is larger than the working rate there. One of the typical weather situations which accord with the above conditions is the warm V-shaped inverted trough which is behind the high travelling over the East China Sea, East Asia during early summer (Fig. 2). In this situation, because of the high temperature and low pressure within the trough, $\delta\rho_d < 0$. It is possible to find out a ρ_d—minimum region with $\nabla\rho_d=0$ in the trough, hence

$$A_{22}\simeq\left[\left(-\frac{\delta \boldsymbol{V}_h}{T}\cdot\nabla_h C_{pd}T\right)-\left(-\frac{\delta \boldsymbol{V}_h}{T}\cdot\nabla_h R_d T\right)\right]\delta\rho_d$$

$$=-\frac{\delta \boldsymbol{V}_h}{T}\cdot(\nabla_h C_v T)\delta\rho_d. \tag{23}$$

As a result, the warm advection behind the high $\left(-\delta v\frac{\partial C_v T}{\partial y}>0\right)$ will make the density of this region continue going down, thus increasing the instability. This is the situation that the regenesis of cyclonic waves often takes place over the warm trough developed and extended eastwards to some extent.

3. *Terms* B_1 *and* B_2

We now discuss the factors dominating the augment of velocity disturbance (δv):

$$(B_1)+(B_2)\equiv-\frac{\delta\rho}{T}\delta u[v(2\Omega\sin\varphi)]+\frac{\delta\rho}{T}\delta v[u(2\Omega\sin\varphi)].$$

These two terms represent the influence of the Coriolis force on the augment of the kinetic energy of disturbances, we will take the blocking high as an example to illustrate the meaning of these two terms (see Fig.3).

The pressure field, as shown in Fig.3 by solid lines, steadily maintains for several days. If a temperature field described by dashed lines emerges, one could expect to have a region (marked by asterisk)with $\delta\rho>0$ over the temperature trough west of the blocking high because of both $\delta T<0$ and $\delta p>0$ there. It can be seen from (B_2)that the southerly disturbance with $\delta v>0$ will be supressed ((B_2)>0) over the asterisk region due to $u>0(v\simeq0)$, thus the flow over the asterisk region tends to regress to zonal circulations. As known, it is just the common evolving characteristics of the temperature-pressure field during the destroying period of the second kind of blocking high (Ye et al., 1962). By the way, the higher the latitude is, the stronger the above-mentioned effect is. As is the case that the blocking high collapse starts often at the higher latitudes and then spreads to the lower ones.

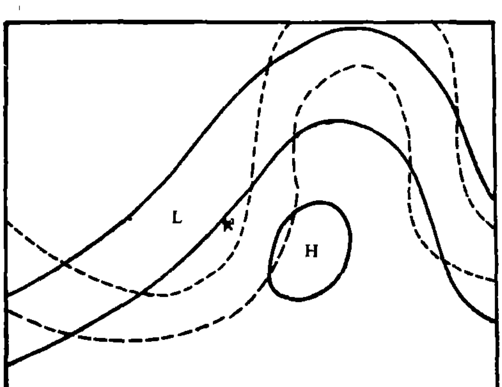

Fig. 3. Discussion of stability for the blocking high tending to destroy.

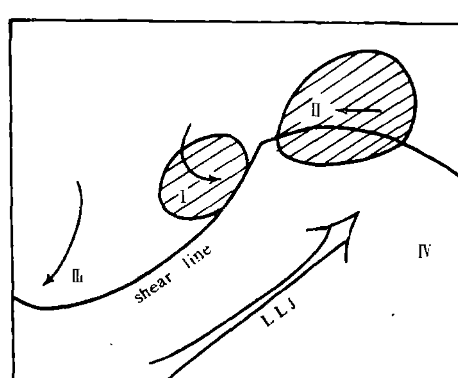

Fig. 4. Discussion of stability of low-level jets.

4. *Terms B_6 and B_7*

$$(B_6)+(B_7)\equiv\frac{1}{T}\frac{\partial v}{\partial x}\delta v\delta(\rho u)+\frac{1}{T}\frac{\partial u}{\partial y}\delta u\delta(\rho v).$$

These two terms represent the influence of the curl of wind field $\left(\frac{\partial v}{\partial x},\ \frac{\partial u}{\partial y}\neq 0\right)$ on the stability. Here, as an example, the conditions of the low-level jet and shear line during the pre-flood period in South China are discussed (see Fig.4 and Table 1). It can be seen in Fig. 4 that (B_6) and (B_7) take negative values simultaneously within regions I and II, while their stability is expected to appear within regions III and IV. The synoptic observations show that regions I and II are just where there often occur heavy rains, especially the heavy rain region in the upper left of the low-level jet (Sun, 1978).

Table 1. The Structure of Stability for a Low-Level Jet

	I	II	III	IV
δu	+	−	−	+
δv	−	+	−	+
$\frac{\partial v}{\partial x}/\frac{\partial u}{\partial y}$	+/+	+/+	+/+	+/+
$(B_6)/(B_7)$	−/−	−/−	+/+	+/+

VI. CONCLUDING REMARKS

This paper has reestablished the concept of Liapunovian direct method and discussed the local stability of a system with emphasis on whether the departure δA from a reference A would grow or not, where the departure is just the so-called disturbance that obeys the same equation system as that for the reference. Mathematically, the (reference) state of minimum departure ($\delta A=0$) corresponds to the trivial solution to the system for disturbances, while physically, the reference is the most probable state. Although the reference state is very definite theoretically, its practical determination is difficult. Alternately, the approximate "most orderless state", e.g., the climate-averaged state over a certain period are usually employed. The further studies are expected to thoroughly solve this problem.

In addition, according to the dissipative structure theory of Prigogine's school the formation of new structures always results from the instability of deterministic branching solutions of the original system. On the other hand, the new structures stem from fluctuations. In other words, it is necessary to use both deterministic and stochastic methods in clear understanding of temporal evolution of the system. For example, the stochastic differential equation theory is just one of the approaches in which we need not test the stability by introducing any perturbations. As soon as a starting distribution is given the temporal evolution could be determined and, also this approach would give the time lag involved in the formation of new states when the system enters into unstable regions. These problems will be discussed in other papers.

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