ON THE EFFECT OF AMBIENT TURBULENCE ON THE BUOYANT PLUME RISE

Du Shuming (杜曙明)

Nanjing Institute of Meteorology, Nanjing 210044

and *Li Zongkai* (李宗恺)

Department of Atmospheric Sciences, Nanjing University, Nanjing 210008

Received September 28, 1992

ABSTRACT

Considering the effects of both entrainment and intensification of the exchanges of momentum and heat between plume and ambient air, we have derived the trajectory equation of buoyant plume under neutral conditions, and the final rise prediction formula theoretically without any hypotheses on the rise termination. Compared with the field experiments, the final rise formula simulates the observed final rise heights well.

Key words: ambient turbulence, plume rise, terminal rise formula, entrainment

I. INTRODUCTION

It is a well–known fact that ambient turbulence is important to the plume rise, and different researchers have different opinions on its mechanism. Using different simplified models, we can get various formulas of the final plume rise.

Most of the plume rise theoretical models use the sudden–effect mechanism, in which the ambient turbulence has no effect on the plume rise at its main stage, but affects it suddenly at a certain point afterwards. This kind of mechanism makes the plume become flat and reach the terminal height within a very short distance. Briggs' (1975) break–up model is a typical one of such kinds of models.

According to the analysis of the plume rise data at home and abroad, ambient turbulence is considered to affect the plume rise at the beginning stage, and its accumulated function can not be ignored. Chen (1981) and Li (1982; 1987) further considered that such kind of accumulated function can be described by enhancing the plume entrainment. They separately computed the entrainment velocity under the effect of ambient turbulence and deduced the plume rise path equation and the final rise formula. This sort of model is called joint–effect model.

In fact, ambient turbulence makes not only the exchange of mass between plume and ambient air, which is called entrainment, but also the exchanges of momentum and heat between ambient air and plume. Considering the function of the latter one, Du and Li (1993) have deduced the trajectory equation of the buoyant plume and the final rise formula under neutral conditions. These theoretical results simulate the observed facts well.

In this paper, with both effects of ambient turbulence included, we deduce the trajectory equation of the plume and the final rise formula under neutral conditions, and compare the results with the observed plume rise data.

II. PLUME RISE EQUATIONS

In this paper, the question of bent–over plume under neutral conditions is considered. According to the continuity equation of fluid dynamics, the whole plume should obey the following relationship of mass conservation (Briggs, 1975)

$$\frac{\mathrm{d}}{\mathrm{d}x}(UR^2)=2RV_e \ , \tag{1}$$

where U is the mean velocity of ambient air, R is the effective radius of plume, x is the downwind distance from stack, and V_e is entrainment velocity.

If the effect of ambient turbulence on entrainment is ignored, generally, we can get the following formula

$$V_e=\beta\overline{W} \ , \tag{2}$$

where β is an empirical constant, as to the buoyant plume, $\beta=0.6$; and $\overline{W}$ is the buoyant velocity of the plume, $\overline{W}=\mathrm{d}z/\mathrm{d}t=U(\mathrm{d}z/\mathrm{d}x)$, of which z is the plume axis height from the exit height of stack.

According to Eqs.(1) and (2), we have

$$R=R_0+\beta z\approx\beta z \ , \tag{3}$$

which is adopted by the sudden–effect model.

Considering the contribution of ambient turbulence to entrainment, we can get the following relationship between plume effective radius R and plume axis height z (Li, 1982)

$$R=\beta z^{1+i} \ , \tag{4}$$

where i is the ambient turbulence intensity. This formula has been verified by the observed data of Xuzhou Power Plant (TGMDNU, 1981).

According to our deduction in which the contribution of ambient turbulence to the exchanges of momention and heat between environment and plume is included, the plume momentum equation and heat equation can be written as

$$\frac{\mathrm{d}F_z}{\mathrm{d}x}=\frac{-2cK_H}{R^2U}F_z \ , \tag{5}$$

$$\frac{\mathrm{d}M_{\mathrm{eff}}}{\mathrm{d}x}=\frac{F_z}{U}-\frac{2cK_m}{R^2U}M_{\mathrm{eff}} \ , \tag{6}$$

where F_z is the plume buoyant flux, M_{eff} is the effective vertical momentum flux of the plume, K_m and K_H are separately the momentum and heat exchange coefficients of the ambient turbulence, and c is an undetermined coefficient. The definitions of F_z and M_{eff} are

$$F_z=\pi^{-1}\int_p gW\frac{\theta'}{\theta_a}\mathrm{d}\sigma \ , \tag{7}$$

and

$$M_{\mathrm{eff}}=\pi^{-1}U\left(\int_p W\mathrm{d}\sigma-\oint\varphi\mathrm{d}y\right) \ , \tag{8}$$

where P is the vertical cross–section of the bent–over plume, θ' is the potential temperature difference between plume and ambient air, θ_a is the potential temperature of ambient air and φ

is the velocity potential of plume.

Considering that only those eddies whose scales are equal to, or even smaller than the radius can play a leading role in the turbulence exchange between plume and ambient air (Li, 1982) and that the plume rise under neutral conditions is just dealt with in this paper, therefore we have, according to Du and Li (1993), the exchange coefficients of turbulent momentum and heat in the form:

$$K_m = K_H = K = 0.1 C_1 R^2 u_{*0} / H_s^{1/2} , \tag{9}$$

where C_1 is an undetermined coefficient, u_{*0} is the surface turbulent frictional coefficient and H_s is the geometric height of stack.

If the contribution of ambient turbulence to the exchanges of momentum and heat is ignored, Eq. (6) can be changed into

$$\frac{\mathrm{d}M_{\mathrm{eff}}}{\mathrm{d}x} = \frac{F_z}{U}. \tag{6$'$}$$

As to passive plume, $F_z = 0$, then (6′) becomes

$$\frac{\mathrm{d}M_{\mathrm{eff}}}{\mathrm{d}x} = 0, \tag{6$''$}$$

M_{eff} has the following relationship with $\overline{W}$ and R which are the vertical velocity and the effective radius of plume:

$$M_{\mathrm{eff}} = R^2 U \overline{W}. \tag{10}$$

Thus, according to (6″), the passive plume should obey the following formula

$$R^2(x) W(x) = R_0^2 W_0,$$

where the subscript "0" represents the value at a certain point x_0. If we choose the source point as the reference point x_0 and notice that the plume radius R and the diffusion coefficient σ (σ_r or σ_z) are nearly equal, we can get

$$\overline{W}(x) \sim \frac{R_0^2}{\sigma^2} W_0.$$

Generally, $\sigma \sim 0.1x$, so the above formula can be changed as

$$\overline{W}(x) \sim 100 \frac{R_0^2}{x^2} W_0.$$

Therefore, due to the effect of initial momentum, the axis rise height of the passive plume in the vertical direction is

$$z \sim \int_{10R_0}^{\infty} \overline{W}(x) \frac{\mathrm{d}x}{U} \sim 10 \frac{W_0}{U} R_0.$$

According to the typical values, $W_0 = 15$ m / s, $U = 5$m / s, $R_0 = 3$ m then $z \sim 90$ m. On the basis of practical experience and observed data, it is impossible that the plume axis can arrive at such a large height only because of the contribution of the initial momentum. So, the term of turbulence exchange in formula (6) can not be ignored. For the same reason, the term of turbulence exchange in formula (5) also has important function.

III. TERMINAL RISE FORMULA

According to Eqs. (5), (6) and (9), we can get the following expression

$$M_{\mathrm{eff}} = (F_m + \frac{F_0}{U}x)\exp(-\frac{0.2C_*}{H_s^{1/2}}\frac{u_{*0}}{U}x), \tag{11}$$

where F_m is the initial momentum flux of plume, F_0 is the initial buoyancy flux of plume, and $C_* = CC_1$.

From formula (4), together with $M_{\mathrm{eff}} = R^2 U\overline{W}$, we have

$$U^2\beta^2 z^{2+2i}\frac{\mathrm{d}z}{\mathrm{d}x} = (F_m + \frac{F_0}{U}x)\exp\left(-\frac{0.2C_*}{H_s^{1/2}}\frac{u_{*0}}{U}x\right). \tag{12}$$

Integrating formula (12) obtains

$$z^{3+2i} = \left(\frac{3+2i}{U^2\beta^2}\right)\left[\frac{F_m}{A}(1-\mathrm{e}^{-Ax}) + \frac{F_0}{A^2U}(1-\mathrm{e}^{-Ax} - Ax\mathrm{e}^{-Ax})\right], \tag{13}$$

where $A = (0.2C_* / H_s^{1/2})(u_{*0} / U)$.

In general, when $x >$ (5–10)$F_m / (F_0 / U)$, the effect of initial momentum can be ignored, thus

$$z^{3+2i} \approx \left(\frac{3+2i}{U^2\beta^2}\right)\frac{F_0}{A^2U}(1-\mathrm{e}^{-Ax} - Ax\mathrm{e}^{-Ax}). \tag{14}$$

When x is very large, the height of plume axis tends to be a finite value, which is usually called the terminal rise height $\triangle H$:

$$\triangle H = \left[\left(\frac{3+2i}{U^2\beta^2}\right)\left(\frac{F_0}{A^2U}\right)\right]^{\frac{1}{3+2i}}, \tag{15}$$

where C_* is a coefficient to be determined. Once it is determined, it can be used to calculate the terminal rise height. Using the twelve sets of observed data obtained in the plume rise experiments of Xuzhou Power Plant, we can determine that C_* is nearly equal to 1.00 (TGMDNU, 1981). Thus, expression (15) can be rewritten as

$$\triangle H = \left[70(3+2i)\left(\frac{F_0}{Uu_{*0}^2}H_s\right)\right]^{\frac{1}{3+2i}} \tag{16}$$

As to a medium–size power plant, for example, if the calculation parameters in Fig. 3 are used, the value of $\triangle H$ at $i=0$ is nearly as large as 1.4 times that of $\triangle H$ at $i=0.1$.

Formula (16) can be verified if the twenty–one sets of observed data obtained in the plume rise tests of Xuzhou Power Plant are adopted. The stack height H_s of this plant is 180m and it can be roughly determined that $i=0.1$ and $U / u_{*0}=12$ when the wind data are used. Fig.1 shows the comparison between observed rise heights and calculated values from formula (16). The correlation coefficient r between observed and calculated values is 0.66, the average value $\overline{K}$ of the ratio of these two values is 1.04, and the standard deviation σ of the ratio is 0.23.

Formula (16) can also be verified when the plume rise data of sixteen foreign power plants (Briggs, 1969) is used. According to the source parameters and meteorological conditions, i is selected as a value of 0.1 and U / u_{*0} is chosen as a value of 14 (Li and Zhu, 1987). Fig.2 shows

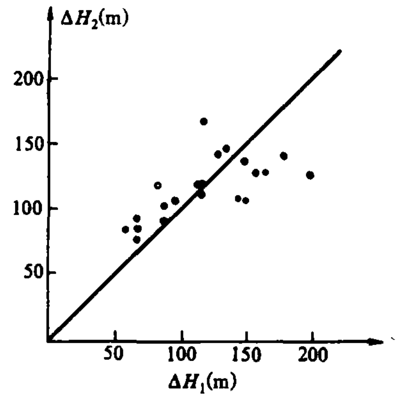

Fig. 1. Comparison between the observed values ($\triangle H_1$) of plume rise tests in Xuzhou Power Plant and the calculated values ($\triangle H_2$) of formula (16).

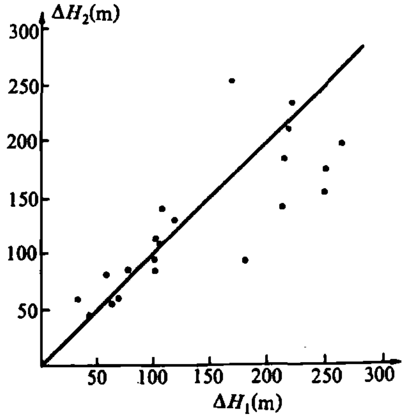

Fig. 2. Comparison of the observed values from the foreign plume rise tests with the calculated values of formula (16).

the comparison between the observed values and calculated ones of plume rise. Because there are no detailed original data, Fig. 2 only gives the comparison under the mode wind in each test. The correlation coefficient r of observed values and calculated ones from the twenty sets of materials is 0.80; $\overline{K}$, the average ratio of these two values, is 1.00 and the standard deviation σ of the ratio is 0.33. After three sets of abnormal data have been rejected, the seventeen selected sets of materials yield $r=0.91$, $\overline{K}=0.96$ and $\sigma=0.27$.

Through the preceding comparison between calculated and observed values from so many plume rise tests whose source parameters and meteorological parameters are all different, we can see that the basic laws shown in formula (16) even which includes C_* determined by observed data, are correct, and consistent with the observed facts, on the whole.

IV. EFFECT OF AMBIENT TURBULENCE

In order to discuss the problem easily, we rewrite expression (14) as

$$z^{3+2\delta_1 i}=\left(\frac{3+2\delta_1 i}{U^2\beta^2}\right)\frac{F_0}{U(\delta_2 A)^2}(1-e^{-\delta_2 Ax}-\delta_2 Axe^{-\delta_2 Ax}), \tag{14'}$$

where δ_1 and δ_2 are indicating factors of ambient turbulence functions. δ_1 is the indicating factor of ambient turbulence contribution to entrainment, the effect of this term is ignored when $\delta_1=0$; δ_2 is the indicating factor of the contribution of ambient turbulence to the exchanges of momentum and heat between plume and ambient air. When $\delta_2=1$, the exchanges of momentum and heat are considered in formula (14′), while they are ignored when $\delta_2=0$.

To begin with, let us talk about the plume rise trajectory equation when $\delta_1=\delta_2=0$, in other words, the effects of ambient turbulence on plume rise are totally ignored. Thus, formula (14′) is simplified as

$$z^3=\left(\frac{3F_0}{\beta^2U^3}\right)\lim_{\delta_2\to 0}\frac{1-e^{-\delta_2 Ax}-\delta_2 Axe^{-\delta_2 Ax}}{(\delta_2 A)^2}=\left(\frac{3}{2\beta^2}\right)\left(\frac{F_0}{U^3}\right)x^2,$$

i.e.

$$z = \left(\frac{3}{2\beta^2}\right)^{1/3} \frac{F_0^{1/3}}{U} x^{2/3}. \tag{17}$$

Here notice that small terms of higher order of $[\delta_2 Ax]^3$ are ignored.

This is the famous "two–thirds law" (Briggs, 1975). Obviously, it is an special case of formula (14′) when $\delta_1 = \delta_2 = 0$.

For the same reason, when $\delta_1 = 1$ and $\delta_2 = 0$, (14′) can be simplified as

$$z = \left(\frac{3+2i}{2\beta^2}\right)^{\frac{1}{3+2i}} \left(\frac{F_0}{U^3}\right)^{\frac{1}{3+2i}} x^{\frac{2}{3+2i}}. \tag{18}$$

This is the "sub–two–thirds law" presented by Li (1982).

From formulas (17) and (18), when the effect of enchancing entrainment by ambient turbulence is considered, the plume trajectory has a systematical deviation from that of the " two–thirds law" which totally ignores the ambient turbulence functions. And the speed of plume rise has reduced to some extent. However, because of ignoring the effect of exchanges of heat and momentum by ambient turbulence, the plume height still rises without any limits with increasing of the downwind distance x.

If we only take account of the contribution of ambient turbulence to the exchanges of momentum and heat while ignoring the effect of enhancing entrainment by ambient turbulence, that is, $\delta_1 = 0$ and $\delta_2 = 1$; therefore, we have (Du and Li, 1993)

$$z = \left(\frac{3}{\beta^2}\right)^{1/3} \left(\frac{F_0}{A^2 U^3}\right)^{1/3} (1 - e^{-Ax} - Axe^{-Ax})^{1/3}, \tag{19}$$

where $A = [0.2 / H_s^{1/2}][u_{*0} / U]$. From this formula, we can get the following conclusions: (1) When x or the ambient turbulent intensity (u_{*0} / U) is very small, the trajectory equation (19) of plume rise is simplified into " two–thirds law". (2) When x is very big, the plume axis height z tends to be a limited height $z_{\max}$, which is usually called the terminal rise height, and $z_{\max} = [3F_0 / (A^2 \beta^2 U^3)]^{1/3}$. Thus it can be seen that including the contribution of ambient turbulence to the exchanges of momentum and heat means drawing the break–up mechanism of plume structure, which makes the plume height no longer rise unlimitedly within a large downwind distance. Therefore, the long–unresolved problem of the rise cut–off scheme in the plume rise research can be solved preliminarily. It also shows that the plume trajectory near sources obeys the "two–thirds law" when the effect of enhancing entrainment by ambient turbulence is ignored. This result has certain deviation from the actual facts (Chen, 1981; Li and Zhu, 1987).

Finally, let us think about such kind of situation, in which the effects of both enhancing the entrainment and intensifying the exchanges of momentum and heat by ambient turbulence are considered, in other words, $\delta_1 = \delta_2 = 1$. From formula (14′), when x is very small, the plume trajectory equation can be simplified into

$$z = \left(\frac{3+2i}{2\beta^2}\right)^{\frac{1}{3+2i}} \left(\frac{F_0}{U^3}\right)^{\frac{1}{3+2i}} x^{\frac{2}{3+2i}}. \tag{18}$$

This result is the same as what we have got when $\delta_1 = 1$ and $\delta_2 = 0$. But it should be pointed out that this formula can only be used within a small downwind distance when $\delta_1 = \delta_2 = 1$. If it is used in a large distance, the result will have a considerable deviation from what is calculated by formula (14′). With increasing of x, the plume axis height z tends to be a definite height z_{max}

$$z_{max} = \left(\frac{3+2i}{\beta^2}\right)^{\frac{1}{3+2i}} \left(\frac{F_0}{A^2 U^3}\right)^{\frac{1}{3+2i}}, \tag{20}$$

where $A = [0.2/(H_s)^{1/2}](u_{*0}/U)$. If each variable gets its typical value: $H_s = 200$ m, $u_{*0}/U = 0.1$, $i = 0.1$, $F_0 = 400$ m / s , and $U = 5$ m / s, we have $z_{max} = 149.4$ m, when $\delta_1 = 0$, $\delta_2 = 1$ and $z_{max} = 111.5$ m, when $\delta_1 = 1$, $\delta_2 = 1$.

Through the analysis of formula (20), it can be concluded that the effect of enhancing entrainment by ambient turbulence influences the plume rise from the beginning to the end. At the beginning of plume rise, such kind of function makes the plume trajectory have a systematical deviation from what is calculated by the classical "two–thirds law", in other words, it causes the slope to become smaller and makes the rise speed become slower; while during the later period of plume rise, its accumulation makes the terminal rise height decrease. However, the influence on the plume rise caused by the exchanges of heat and momentum due to ambient turbulence is obviously different at each stage of the plume rise. At the beginning of plume rise, such kind of effect has nearly no influence on the plume rise, which can be got from the trajectory equation of a short distance, while with increasing plume height, it makes the plume momentum and heat be transported to the ambient air continuously, which finally causes the rise–driving factors to be exhausted.

In order to have a more direct view of the influence of ambient turbulence on plume rise, the preceding analysis has been synthesized in Fig. 3. The calculation parameters used in this figure are: $F = 400$m / s, $H_s = 200$m / s, $U = 5$m / s, $U / u_{*0} = 14$ and $i = 0.1$.

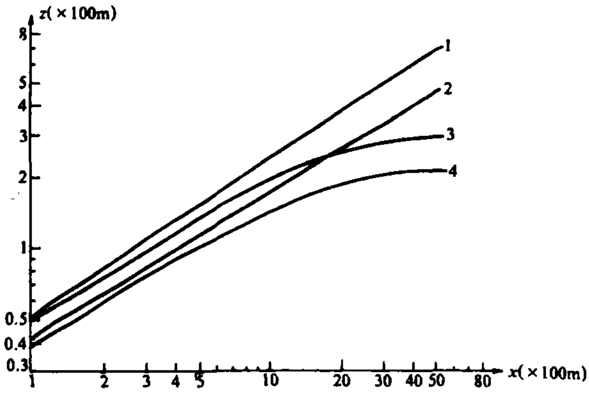

Fig. 3. Effect of ambient turbulence on plume trajectory and terminal rise height. 1: $\delta_1 = 0$, $\delta_2 = 0$; 2: $\delta_1 = 1$, $\delta_2 = 0$; 3: $\delta_1 = 0$, $\delta_2 = 1$; 4: $\delta_1 = 1$, $\delta_2 = 1$.

V. CONCLUSIONS AND COMMENTS

In this paper, considering the effects of ambient turbulence on plume rise through enhancing the entrainment and intensifying the exchanges of momentum and heat between plume and ambient air, we have deduced the trajectory equation of buoyant plume under neutral conditions. Because of having included the function of the latter factor, the terminal rise formula can be derived directly from the trajectory equation without any hypotheses on the rise termination, which can solve the problem of terminal rise theoretically. Through the calculation of actual examples, it shows that the terminal rise formula deduced in this paper is considerably consistent with the observed data in the plume rise tests at home and abroad.

The analysis of influential factors on the plume rise by ambient turbulence shows that the two mechanisms of enhancing entrainment and intensifying the exchanges of momentum and heat are all important to the plume rise, of which, the former has distinctive effect on the plume rise from the beginning to the end while the latter one has a phased character, that is, it only has clear influence in the long distance.

It should be pointed out, in the paper, that the terminal mechanism has been brought in at the very beginning of plume rise and the terminal phase has been considered as a continuous one, through which the problem of choosing the terminal scheme can be avoided, and that such kind of treatment is more close to the actual facts. However, this method has the problem of how to determine the parameters. The terminal height in this paper is the one at which the plume has totally become flat, while the observed rise height is very strongly dependent on the measuring methods and the defination of the terminal rise. According to this model, using longer plume trajectory to determine C_* will be better, but it needs higher quality data observed of plume trajectory. Therefore, the more accurate value of C_* can be obtained through more observed data in future.

REFERENCES

Briggs, G. A. (1969), Plume rise, *USAEC Critical Review Series*, TID–25075.

Briggs, G. A. (1975), Plume rise prediction, *Lectures on Air Pollution and Environmental Impact Analyses* (Ed. D. A. Haugen), AMS, USA.

Chen Jiayi (1981), Effect of ambient turbulence on the plume rise, *Acta Meteor. Sin.*, **39**: 59—69 (in Chinese).

Du Shuming and Li Zongkai (1993), An analytical model of the buoyant plume rise (will be published).

Li Zongkai (1982), A plume rise trajectory equation considering the effects of ambient turbulence, *Acta Meteor. Sin.*, **40**: 80—88 (in Chinese).

Li Zongkai and Zhu Xuanxuan (1987), A model of the buoyant plume rise limited by the mechanical turbulence, *Chinese Science* (B), **7**: 780—788 (in Chinese).

Turbulence Group of Meteor. Dept. of Nanjing Univ. (1981), Experimental research on the plume rise and diffusion rules of a power plant, *J. Envir. Sci.*, **1**: 31—40 (in Chinese).